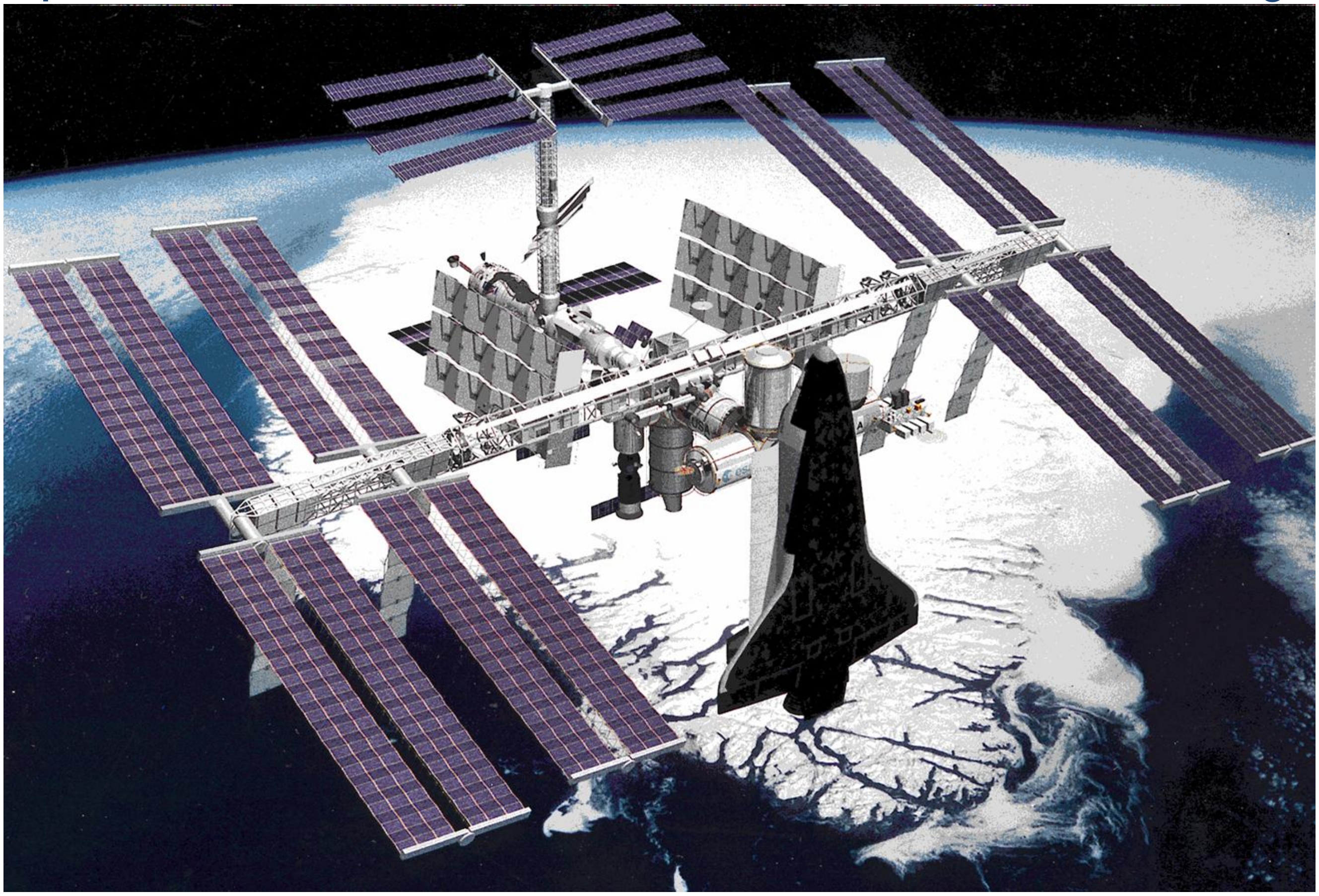
Space Station Attitude and Momentum Control Design



In this example we design a control system for a large and flexible Space Station that is in orbit around the earth. The Space Station consists of a truss structure with some attached modules for the crew, equipment, experiments, etc. which are located near the center of the structure. The attitude control system uses reaction control jets (RCS) and control moment gyros (CMG) but in this example we will examine only CMG control and design a system that stabilizes the Station attitude and manages the CMG momentum from saturating. The Space Station orbit is circular and its attitude is almost constant relative to the Local Vertical Local Horizontal (LVLH) frame. The LVLH x-axis is in the direction of the velocity vector, the z-axis is pointing towards the earth center and the y-axis towards the right solar array.

The Attitude Control System (ACS) has different modes of operation and one of the control modes is the TEA seeking mode where the spacecraft attitude converges to one of the Torque Equilibrium Attitudes (TEA). This happens when the ACS balances the average aerodynamic torques with the gravity gradient torques. The CMGs are considered as a cluster located near the center and not modeled individually. They are momentum exchange devices that have limited torque and momentum. Momentum is the integral of torque, that is, they can only supply torque for a limited time before they saturate and when they do they require momentum desaturation. Momentum dump is achieved either by firing RCS jets or by applying gravity gradient torques. The ACS manages the CMG momentum by positioning the spacecraft attitude at the TEA, and therefore, without firing the RCS jets. When operating in this mode the Station converges to the average TEA and avoids secular CMG momentum build up. When the momentum begins to grow in a certain direction due to external disturbances, the ACS changes the attitude in the direction that reduces momentum.

The Station has a horizontal boom with two rotating solar arrays which are always pointing towards the sun, completing, therefore, one rotation per orbit relative to the spacecraft. The aerodynamic disturbances cause the CMG momentum to cycle but the control system prevents it from reaching saturation. The aerodynamic disturbances consist of steady torques and also cyclic components that excite the spacecraft attitude to oscillations. There are two frequency components associated with the cyclic disturbance torques: one is at orbital rate (ω_o) due to the difference in atmospheric density between the sunny and the dark sides of the earth, and the second component is at twice the orbital rate ($2\omega_o$) caused by drag variation due to the solar arrays rotation. In this mode of operation the function of the CMG control system is not to maneuver the Space Station attitude but to stabilize it at the TEA and to attenuate the attitude oscillations which are caused by cyclic aerodynamic disturbances.

The purpose of this example is to familiarize the analyst with the Flixan program, create rigid and flexible spacecraft models and use them for control design. We will develop linear and non-linear rigid and flexible spacecraft models, design LQR state-feedback for attitude control and momentum management. Since the spacecraft is stabilized in the LVLH frame the model attitude and rates is calculated in the LVLH frame. We will also analyze the system's stability and performance by using simulations and frequency response analysis.

1. Dynamic Model

The linear dynamic model in Equation 1 calculates the spacecraft attitude, rate and CMG momentum relative to the LVLH frame. The linearized gravity gradient torque is also included in the model. It is a function of attitude (ϕ,θ,ψ) and therefore it can be used to regulate the CMG momentum by adjusting the station attitude. The spacecraft states are: LVLH attitude, LVLH rates, and CMG momentum in body axes. The CMG momentum integral is also included to help bound the CMG momentum. The inputs are the CMG control torques T_c and external disturbances T_d. It also includes the linearized gravity gradient dynamics which are functions of the LVLH Euler angles.

$$\begin{bmatrix} I_{XX} & I_{XY} & I_{XZ} \\ I_{XY} & I_{YY} & I_{YZ} \\ I_{XZ} & I_{YZ} & I_{ZZ} \end{bmatrix} \begin{pmatrix} \ddot{\phi} \\ \ddot{\theta} \\ \ddot{\psi} \end{pmatrix} = \omega_o \begin{bmatrix} 0 & 2I_{YZ} & I_{XX} - I_{YY} + I_{ZZ} \\ -2I_{YZ} & 0 & 2I_{XY} \\ -I_{XX} + I_{YY} - I_{ZZ} & -2I_{XY} & 0 \end{bmatrix} \begin{pmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{pmatrix}$$

$$+ \omega_o^2 \begin{bmatrix} 4(I_{ZZ} - I_{YY}) & 3I_{XY} & -I_{XZ} \\ 4I_{XY} & 3(I_{ZZ} - I_{XX}) & I_{YZ} \\ -4I_{XZ} & -3I_{YZ} & (I_{XX} - I_{YY}) \end{bmatrix} \begin{pmatrix} \phi \\ \theta \\ \psi \end{pmatrix} + \omega_o^2 \begin{pmatrix} -4I_{YZ} \\ 3I_{XZ} \\ I_{XY} \end{pmatrix} + T_d + T_c$$

$$\begin{pmatrix} \dot{h}_X \\ \dot{h}_Y \\ \dot{h}_Z \end{pmatrix}_{CMG} = \omega_o \begin{pmatrix} h_Z \\ 0 \\ -h_X \end{pmatrix}_{CMG} - T_c$$

Equation 1 Linearized Spacecraft Dynamic Equations with Attitude in the LVLH frame

2. Design Concepts

At the torque equilibrium attitude (TEA) the CMG momentum is cyclic but it does not diverge to saturation. We can, therefore, design a control system that converges the Space Station attitude to the TEA, which is a stable position because the average aerodynamic torque balances with the average gravity gradient torque. This is accomplished by applying feedback from the CMG momentum to prevent it from diverging and by this process the system attitude converges to the TEA. The LQR control method will be used to design the state-feedback gain that stabilizes the coupled dynamic system. It optimizes a performance index that is defined by the state and control weight matrices Qc and Rc. The LQR method requires a linear dynamic model to synthesize the controller and the Flixan design model described in Equation 1 will be used as a synthesis model. The state feedback control law, therefore, stabilizes not only the attitude but also the CMG momentum by keeping it oscillating around zero. It prevents it from diverging to saturation and as a result the spacecraft attitude converges to the TEA. The CMG momentum, however, is not unstable but it is cycling because it responds to the cyclic aerodynamic disturbances. This is a continuous momentum desaturation method which is very attractive because it does not require RCS propellant. It adjusts the spacecraft attitude and uses gravity gradient to prevent the CMG momentum from building up. It relies on sufficient knowledge of the vehicle mass properties for the derivation of the control gains.

In the following sections will present two designs: a simple state-feedback design, and a more complex design that includes disturbance attenuation filters. We will analyze both designs and compare results.

3. Simple Design without Filters

We begin with a preliminary state-feedback design that is based on a dynamic model derived from Equation 1. We will use Flixan to create the rigid and flexible spacecraft models, design the control system using LQR, and analyze stability.

3.1 Flixan Models

The files for this analysis are in directory: *"Flixan\Control Analysis\LQG\Examples\Space-Station w CMG2\Design-1"*. The Space Station parameters are defined in the input file "*Space_Station.Inp*" which includes two flight vehicle datasets: a rigid-body "*Space Station with Double-Gimbal CMG Array (Rigid)*", and a flexible model with 34 structural modes "*Space Station with Double-Gimbal CMG Array (Flex)*". The modes are already selected and scaled and they are located in dataset "*Space Station with Double-Gimbal CMG Array, 34 Flex Modes*". The "**LVLH Attitude & Rate**" flag is included in the flags line that defines the output attitude and rate relative to the LVLH frame. It uses the -0.063 (rad/sec) pitch rate which is equal to the orbital rate ω_0 for the transformation. A batch set is included at the top of the input file with title "*Batch for Large Flexible Space Station*". It is shown below and it is used for processing the entire input file fast.

```
BATCH MODE INSTRUCTIONS ...............
Batch for Large Flexible Space Station
! This batch set creates a model for a Space Station that is
! controlled by an array of double-gimbal CMGs. Two models
! are created, a rigid-body model and a flexible model using the
! attached modal data. The design model is extracted from Rigid Vehicle
! and the plant state is transformed so that it is equal to the Output, C=I
!
Retain Matrix    : State Weight Matrix Qc (12x12)
Retain Matrix    : Control Weight Matrix Rc (3x3)
!
Flight Vehicle   : Space Station with Double-Gimbal CMG Array (Rigid)
Flight Vehicle   : Space Station with Double-Gimbal CMG Array (Flex)
System Modificat : Space Station with Double-Gimbal CMG Array (Design Plant)
System Modificat : Space Station with Double-Gimbal CMG Array (LVLH Plant)
Transf-Function  : 3 Integrators
System Connection: Augmented Design Plant
LQR Control Des  : LQR Control Design for Augmented Space Station Model
!
To Matlab Format : Space Station with Double-Gimbal CMG Array (Rigid)
To Matlab Format : Space Station with Double-Gimbal CMG Array (Flex)
To Matlab Format : Space Station with Double-Gimbal CMG Array (Design Plant)
To Matlab Format : Space Station with Double-Gimbal CMG Array (LVLH Plant)
To Matlab Format : Augmented Design Plant
To Matlab Format : LQR State-Feedback Gain for Augmented Design Plant
-------------------------------------------------------------------------------------------------
```

```
FLIGHT VEHICLE INPUT DATA ......
Space Station with Double-Gimbal CMG Array (Rigid)
!
! The Space Station state-space model is created using the vehicle modeling program.
! The model uses an array of double-gimbal control moment gyros, 3 rate
! gyros, 3 attitude sensors, and 4 accelerometers. The Station is initialized at the
! Local Vertical Local Horizontal (LVLH) attitude and it has a negative pitch rate
! -0.063 radians/sec which is equal to the orbital rate. The vehicle rates are with
! respect to the LVLH frame. A constant bias torque 7.40153 (ft-lb) is applied
! in the direction (-0.1969, 0.5963, 0.7781) to represent the steady-state gyroscopic
! and gravity-gradient torques due to the constant pitch rate
!
Body Axes Output, LVLH Attitude & Rate

Vehicle Mass (lb-sec^2/ft), Gravity Accelerat. (g) (ft/sec^2), Earth Radius (Re) (ft)    :   6200.0   32.174
Moments and products of Inertias Ixx, Iyy, Izz, Ixy, Ixz, Iyz, in (lb-sec^2-ft)          :   0.1194e+9  0.404e+8
CG location with respect to the Vehicle Reference Point, Xcg, Ycg, Zcg, in (feet)        :   0.0          0.0
Vehicle Mach Number, Velocity Vo (ft/sec), Dynamic Pressure (psf), Altitude (feet)       :   0.0          25500.0
Inertial Acceleration Vo_dot, Sensed Body Axes Accelerations Ax,Ay,Az (ft/sec^2)         :   0.0          0.0
Angles of Attack and Sideslip (deg), alpha, beta rates (deg/sec)                         :   0.0          0.0
Vehicle Attitude Euler Angles, Phi_o,Thet_o,Psi_o (deg), Body Rates Po,Qo,Ro (deg/sec)   :   0.0000       0.000
W-Gust Azim & Elev angles (deg), or Torque/Force direction (x,y,z), Force Locat (x,y,z)  :   Torque    0.1969  -0.5963
Surface Reference Area (feet^2), Mean Aerodynamic Chord (ft), Wing Span in (feet)        :   0.0    1.0    1.0
Aero Moment Reference Center (Xmrc,Ymrc,Zmrc) Location in (ft), {Partial_rho/ Partial_H} :   0.0          0.0
Aero Force Coef/Deriv (1/deg), Along -X, {Cao,Ca_alf,PCa/PV,PCa/Ph,Ca_alfdot,Ca_q,Ca_bet}:   0.0          0.0
Aero Force Coeffic/Derivat (1/deg), Along Y, {Cyo,Cy_bet,Cy_r,Cy_alf,Cy_p,Cy_betdot,Cy_V}:   0.0         -0.0
Aero Force Coeff/Deriv (1/deg), Along Z, {Czo,Cz_alf,Cz_q,Cz_bet,PCz/Ph,Cz_alfdot,PCz/PV}:   0.0         -0.0
Aero Moment Coeffic/Derivat (1/deg), Roll: {Clo, Cl_beta, Cl_betdot, Cl_p, Cl_r, Cl_alfa}:   0.0         -0.0
Aero Moment Coeff/Deriv (1/deg), Pitch: {Cmo,Cm_alfa,Cm_alfdot,Cm_bet,Cm_q,PCm/PV,PCm/Ph}:   0.0         -0.0
Aero Moment Coeffic/Derivat (1/deg), Yaw : {Cno, Cn_beta, Cn_betdot, Cn_p, Cn_r, Cn_alfa}:   0.0          0.0

Number of External Torques on the Vehicle                                                :  3
Torque No  1  Direction (x, y, z)                                                        :  1.0       0.0       0.0
Torque No  2  Direction (x, y, z)                                                        :  0.0       1.0       0.0
Torque No  3  Direction (x, y, z)                                                        :  0.0       0.0       1.0

Double Gimbal Control Moment Gyro System  (3-axes), Initial Momentum (x,y,z) (ft-lb-sec) :  Yes    0.0    0.0    0.0

Number of Gyros, (Attitude and Rate)                                                     :  6
Gyro No  1 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Locat,   Node 2   :  Roll    Rate  0.0    82.0
Gyro No  2 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Locat,   Node 2   :  Pitch   Rate  0.0    82.0
Gyro No  3 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Locat,   Node 2   :  Yaw     Rate  0.0    82.0
Gyro No  4 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Locat,   Node 2   :  Roll    Attitu  0.0 82.0
Gyro No  5 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Locat,   Node 2   :  Pitch   Attitu  0.0 82.0
Gyro No  6 Axis:(Pitch,Yaw,Roll), (Attitude, Rate, Accelerat), Sensor Locat,   Node 2   :  Yaw     Attitu  0.0 82.0

Number of Accelerometers, Along Axes: (x,y,z)                                            :  4
Acceleromet No  1 Axis:(X,Y,Z), (Position, Velocity, Acceleration), Sensor Loc, Node 2   :  X-axis Accelerat.  0.0
Acceleromet No  2 Axis:(X,Y,Z), (Position, Velocity, Acceleration), Sensor Loc, Node 2   :  Z-axis Accelerat.  0.0
Acceleromet No  3 Axis:(X,Y,Z), (Position, Velocity, Acceleration), Sensor Loc, Node 4   :  X-axis Accelerat.  0.0
Acceleromet No  4 Axis:(X,Y,Z), (Position, Velocity, Acceleration), Sensor Loc, Node 4   :  Z-axis Accelerat.  0.0

Number of Bending Modes                                                                  :  0
-----------------------------------------------------------------------------------------------
```

The input file includes a system modification dataset "*Space Station with Double-Gimbal CMG Array (Design Plant)*" which creates the control design model by extracting a reduced number of inputs, states, and outputs from the rigid model. The systems and matrices created are saved in file "*Space_Station.Qdr*" under the same title.

```
CREATE A NEW SYSTEM FROM AN OLD SYSTEM... (Titles of the New and Old Systems)
Space Station with Double-Gimbal CMG Array (Design Plant)
Space Station with Double-Gimbal CMG Array (Rigid)
! Create the Station Design Model by Reducing the Rigid-Body Model
! Inputs are the CMG Control Torques
! Outputs are: LVLH attitude and rates, and CMG Momentum
!
TRUNCATE OR REORDER THE SYSTEM INPUTS, STATES, AND OUTPUTS
Extract Inputs :   1    2    3
Extract States :   1    3    5    2    4    6   11   12   13
Extract Outputs:   1    3    5    2    4    6   16   17   18
-----------------------------------------------------------------------------------------------
```

An additional transformation is needed because although the design plant attitude and rates are in the LVLH frame, some of the states, however, are still in the body frame. Since we assume that the state-feedback measurements are in the LVLH we want our design model state to be also in the LVLH frame. This transformation makes the design plant states to be equal to the outputs which are already LVLH attitudes and rates, and the output matrix C=Identity. The dataset below performs this transformation. Its title is "*Space Station with Double-Gimbal CMG Array (LVLH Plant)*". Notice that the transformation requires the design plant matric C to be square and observable.

```
CREATE A NEW SYSTEM FROM AN OLD SYSTEM... (Titles of the New and Old Systems)
Space Station with Double-Gimbal CMG Array (LVLH Plant)
Space Station with Double-Gimbal CMG Array (Design Plant)
! Transform the design plant and make the States equal to the Outputs
SYSTEM TRANSFORMATION, STATES EQUAL TO OUTPUTS
---------------------------------------------------------------------------------------------------------

SYSTEM OF TRANSFER FUNCTIONS ...
3 Integrators
! Used to calculate the CMG Momentum Integral for the LQR Optimization
Continuous
TF. Block #  1  Integrator x                                      Order of Numer, Denom=  0  1
Numer 0.0         1.0
Denom 1.0         0.0
TF. Block #  2  Integrator y                                      Order of Numer, Denom=  0  1
Numer 0.0         1.0
Denom 1.0         0.0
TF. Block #  3  Integrator z                                      Order of Numer, Denom=  0  1
Numer 0.0         1.0
Denom 1.0         0.0
.....................................................
Block #, from Input #, Gain
      1             1       1.0
      2             2       1.0
      3             3       1.0
...........................
Outpt #, from Block #, Gain
      1             1       1.0
      2             2       1.0
      3             3       1.0
...........................
Definitions of Inputs  =   3
CMG Momentum-X
CMG Momentum-Y
CMG Momentum-Z

Definitions of Outputs =   3
CMG Momentum-X Integral
CMG Momentum-Y Integral
CMG Momentum-Z Integral
------------------------------------------------------------------------------
```

The design model is finally augmented by including 3 additional states, the (x, y, z) momentum integral because it is not sufficient to only bound the momentum from diverging but we also want it to be cycling near zero without a bias. The 3 momentum integrators are implemented as 3 transfer functions using the transfer functions combination utility. It creates the state-space system "*3 Integrators*", shown above. We must also create the "*Augmented Design Plant*" by combining the LVLH design plant with the 3 additional (x, y, z) momentum integral states using the Flixan systems combination utility, see Fig.2.

```
INTERCONNECTION OF SYSTEMS .....
Augmented Design Plant
! Augment the Design Plant by including the Momentum Integral and the
! Oscillation Filters
Titles of Systems to be Combined          (Found in File Comb_tst.Qdr)
Title 1 Space Station with Double-Gimbal CMG Array (LVLH Plant)
Title 2 3 Integrators
SYSTEM INPUTS TO SUBSYSTEM  1
Via Matrix +I3                                                                      3 CMG Control Torques
..............................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  1                                                    9 Plant States
System Output  1 from Subsystem  1, Output  1, Gain= 1.0                            3 LVLH Attitudes
System Output  2 from Subsystem  1, Output  2, Gain= 1.0
System Output  3 from Subsystem  1, Output  3, Gain= 1.0
System Output  4 from Subsystem  1, Output  4, Gain= 1.0                            3 LVLH Rates
System Output  5 from Subsystem  1, Output  5, Gain= 1.0
System Output  6 from Subsystem  1, Output  6, Gain= 1.0
System Output  7 from Subsystem  1, Output  7, Gain= 1.0                            3 CMG Momentum
System Output  8 from Subsystem  1, Output  8, Gain= 1.0
System Output  9 from Subsystem  1, Output  9, Gain= 1.0
..............................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  2                                                    3 Momentum Integrals
System Output 10 from Subsystem  2, Output  1, Gain= 1.0                            H-integr-X
System Output 11 from Subsystem  2, Output  2, Gain= 1.0                            H-integr-Y
System Output 12 from Subsystem  2, Output  3, Gain= 1.0                            H-integr-Z
..............................................................................
SUBSYSTEM NO  1 GOES TO SUBSYSTEM NO  2                                             to Momentum Integrat
Subsystem  1, Output  7 to Subsystem  2, Input  1, Gain=  1.0                       CMG Momentm-X
Subsystem  1, Output  8 to Subsystem  2, Input  2, Gain=  1.0                       CMG Momentm-Y
Subsystem  1, Output  9 to Subsystem  2, Input  3, Gain=  1.0                       CMG Momentm-Z
......................................................................
Definitions of Inputs  =   3
Roll CMG Momentum (ft-lb)
Ptch CMG Momentum (ft-lb)
Yaw  CMG Momentum (ft-lb)

Definitions of Outputs =  12
Roll Attitude  (phi-LVLH)  (radians)
Pitch Attitude (thet-LVLH) (radians)
Yaw  Attitude  (psi-LVLH)  (radians)
Roll Rate      (p-lvlh)    (rad/sec)
Pitch Rate     (q-lvlh)    (rad/sec)
Yaw  Rate      (r-lvlh)    (rad/sec)
CMG Momentum in X-axis (ft-lb-sec)
CMG Momentum in Y-axis (ft-lb-sec)
CMG Momentum in Z-axis (ft-lb-sec)
CMG Momentum Integral X (ft-lb-sec^2)
CMG Momentum Integral Y (ft-lb-sec^2)
CMG Momentum Integral Z (ft-lb-sec^2)
-------------------------------------------------------------------------------------
```

The final dataset included in the input file is the "*LQR Control Design for Augmented Space Station Model*" which calculates the (3x12) state-feedback matrix K_{pqr}. The state and control weight matrices Qc and Rc are already included in file "*Space_Station.Qdr*". The state-feedback matrix K_{pqr} is also saved in file "*Space_Station.Qdr*" and its title is "*LQR State-Feedback Gain for Augmented Design Plant*".

```
LINEAR QUADRATIC REGULATOR STATE-FEEDBACK CONTROL DESIGN
LQR Control Design for Augmented Space Station Model
Plant Model Used to Design the Control System from:      Augmented Design Plant
Criteria Optimization Output is Matrix C
State Penalty Weight (Qc) is Matrix:   Qc12              State Weight Matrix Qc (12x12)
Control Penalty Weight (Rc) is Matrix: Rc3               Control Weight Matrix Rc (3x3)
Continuous LQR Solution Using Assymptotic Method
LQR State-Feedback Control Gain Matrix Kpqr              LQR State-Feedback Gain for Augmented Design Plant
--------------------------------------------------------------------------------------------------------------
```

Six Matlab conversion datasets are also included in "*Space_Station.Inp*". They convert the systems and matrices in m-file format that can be loaded into Matlab by running the script file start.m.

```
CONVERT TO MATLAB FORMAT ........      (Title, System/Matrix, m-filename)
Space Station with Double-Gimbal CMG Array (Rigid)
System
vehicle_rigid
----------------------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........      (Title, System/Matrix, m-filename)
Space Station with Double-Gimbal CMG Array (Flex)
System
vehicle_flex34
----------------------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........      (Title, System/Matrix, m-filename)
Space Station with Double-Gimbal CMG Array (Design Plant)
System
design_plant
----------------------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........      (Title, System/Matrix, m-filename)
Space Station with Double-Gimbal CMG Array (LVLH Plant)
System
lvlh_plant
----------------------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........      (Title, System/Matrix, m-filename)
Augmented Design Plant
System
augm_plant
----------------------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........      (Title, System/Matrix, m-filename)
LQR State-Feedback Gain for Augmented Design Plant
Matrix Kpqr
----------------------------------------------------------------------------------------------------
```

```
[Av, Bv, Cv, Dv]= vehicle_rigid;          % Load the Rigid Vehicle State-Space Model
[Af, Bf, Cf, Df]= vehicle_flex34;         % Load the Flex  Vehicle State-Space Model
[Ad, Bd, Cd, Dd]= design_plant;           % Load the Rigid Design Plant Model
[Al, Bl, Cl, Dl]= lvlh_plant;             % Load the LVLH  Design Plant Model
[Ag, Bg, Cg, Dg]= augm_plant;             % Load the Augmented Design Plant Model
load Kpqr -ascii;                         % Load the LQR gains from previous design
```

The files "vehicle_rigid.m" and "vehicle_flex34.m" are used in simulations. The augmented plant "augm_plant.m" can also be used for the LQR design using Matlab.

3.2 LQR Control System Design

The initialization file "*start.m*" loads the dynamic systems and matrices for the simulation and analysis models. In addition to Flixan, the Matlab script file "*des.m*" is also able to calculate the LQR control gains. It is using either the augmented design plant or by linearizing the Simulink model "*Augm_Vehicle.mdl*" shown in Figure 2. Figure 3 shows the rigid and flexible simulation models: "*Sim_Lin_TEA_Rigid.mdl*" and "*Sim_Lin_TEA_Flex.mdl*", which are similar. They include the Flixan generated systems: "vehicle_rigid.m" and "vehicle_flex34.m" respectively, the second of which contains the 34 structural modes. The rate states of these systems have not been transformed to LVLH as in the design plant. In the flex simulation model, the rate gyro measurements are body rates and they include structural flexibility. A transformation is therefore used, shown in Fig.4, to convert the rate signals to LVLH rates for feedback. The transformation requires the LVLH attitude and the orbital rate ω_0=0.0011 (rad/sec). The model also includes the CMG dynamics implemented as second order transfer functions and the aerodynamic disturbance torques in roll, pitch, and yaw. There is also a bias torque as shown in Equation 1.

```
% LQR Design for Gravity Gradient Momentum Management --------
start;
[Al, Bl, Cl, Dl]= lvlh_plant;                 % Load the LVLH Design Plant Model
[Ag, Bg, Cg, Dg]= augm_plant;                 % Load the Augmented Design Plant Model
[A0,B0,C0,D0]=linmod('Augm_Vehicle');         % Create State-Space system for ...
sys=ss(A0,B0,C0,D0);                          % Analyze the Augmented System
sys=ss(Ag,Bg,Cg,Dg);                          % Analyze the Augmented System
Q0=diag([100,  1.e+6,  100, ...               % lvlh attitude weights
         0.1,  0.1,    0.1, ...               % lvlh rate weights
         1.e-10, 1.e-11, 1.e-10, ...          % CMG Momentum weights
         1.e-12, 3.e-14, 1.e-12]);            % CMG Moment-Integral weights
R0=diag([0.3, 1, 0.1]*1.e-3);                 % CMG Control torque Weights
[Kpqr,S,E] = lqr(sys,Q0,R0)
save Kpqr.mat Kpqr -ascii                     % Save the LQR gains in Kpqr.mat
[A1,B1,C1,D1]=linmod('Simple_Sim'); eig(A1)   % Create State-Space system for ...
```

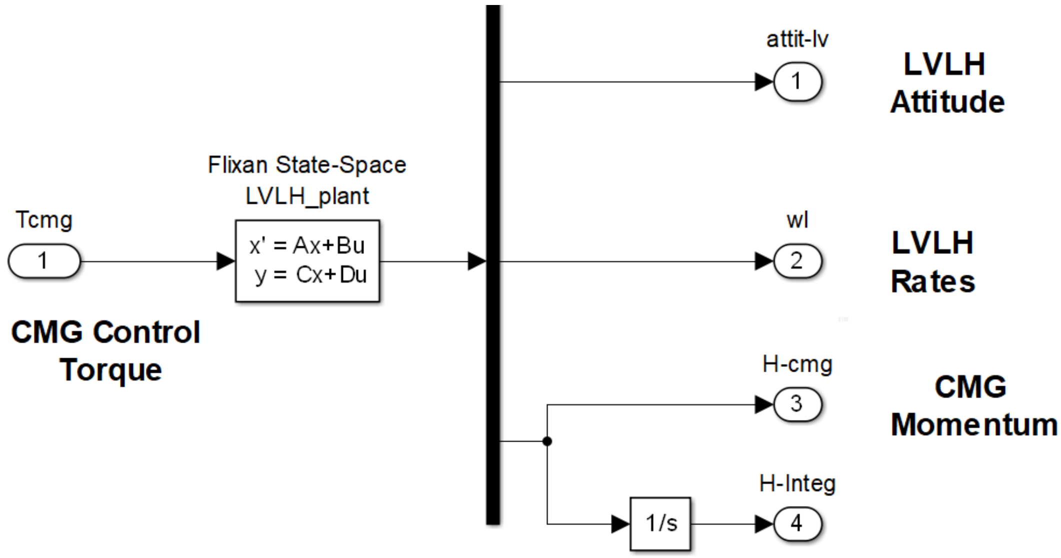


Figure 2 Augmented 12 State Plant Model used for LQR Design including the Momentum Integral

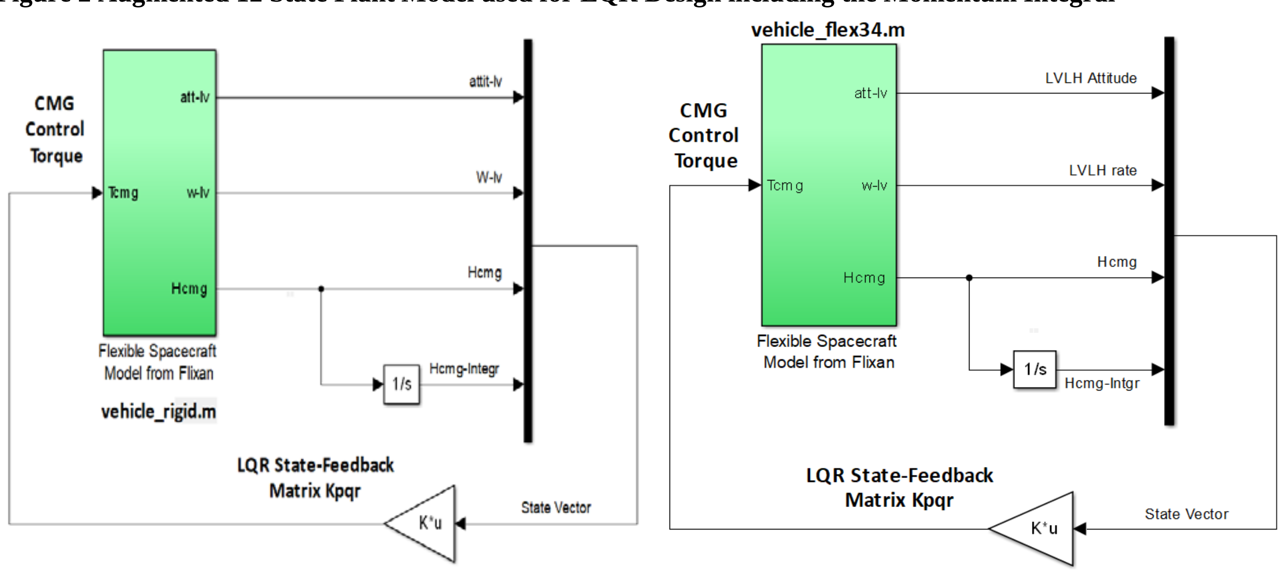


Figure 3 Rigid and Flex Closed Loop Simulation Models with State-Feedback K_{pqr}

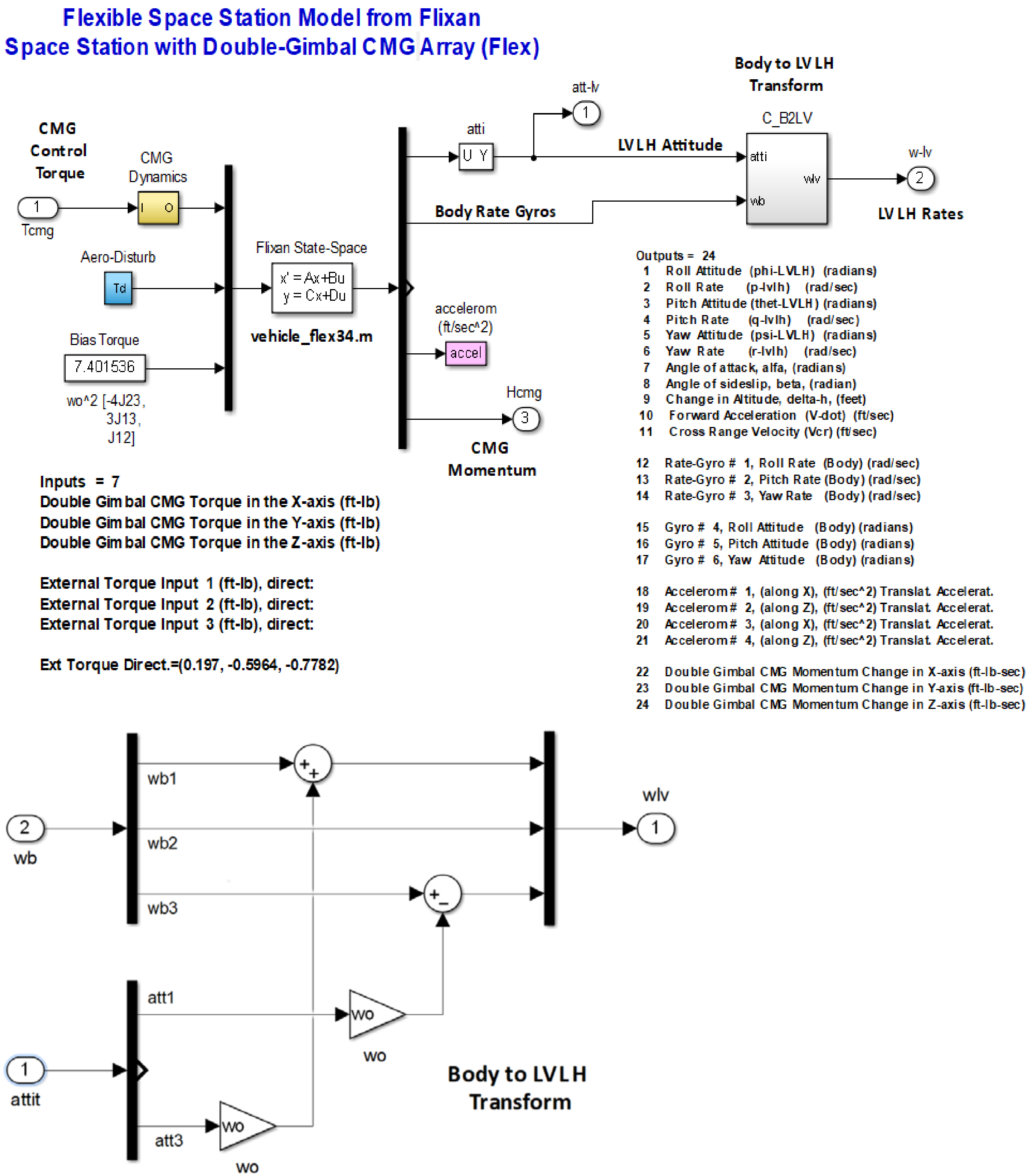
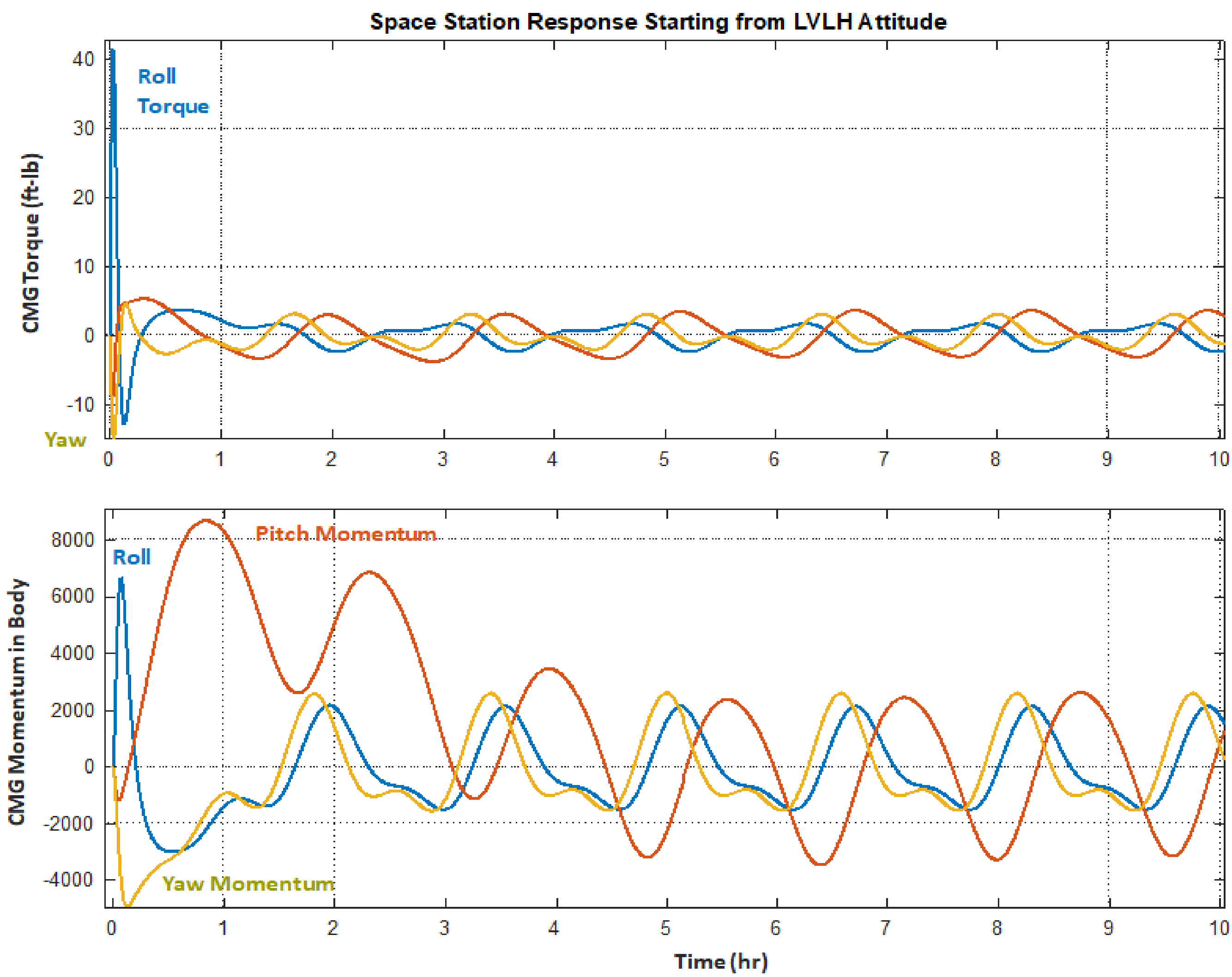


Figure 4 Flexible Spacecraft Subsystem with Body to LVLH Rate Transformation

3.3 Simulation Results

We will now use the simulation model "*Sim_Lin_TEA_Flex.mdl*" to calculate the spacecraft response to aerodynamic disturbances. The disturbance torques are shown in Figure 5 and they consists of cyclic and steady components. They are included in the Space Station dynamics block, shown in Figure 4. There is no attitude command in the system when the Station is operating in this mode. The Space Station attitude is initialized at zero, that is, equal to the LVLH attitude, and it simply drifts under the influence of the external aerodynamic torques and also the gravity gradient torques which are included in the equations of motion.

As the momentum begins to grow the spacecraft changes its attitude, drifting towards torque equilibrium (TEA) and it uses gravity-gradient to balance the steady aero moments. The cyclic components of the disturbances, however, generate attitude oscillations, mostly in-plane (pitch). The state-feedback control stabilizes not only the attitude but also manages the CMG momentum and prevents it from diverging. The momentum integral feedback prevents it from being biased, and it is eventually cycling around zero because of the disturbances at ω_0 and at $2\omega_0$. Notice the control system bandwidth cannot be sufficiently opened in order to provide more torque and to reduce the attitude oscillations caused by the disturbances. The CMGs have torque and momentum limitations and the flex modes can be amplified to instability. In the next section we shall include disturbance accommodation filters to further attenuate the attitude oscillations at orbital rate.



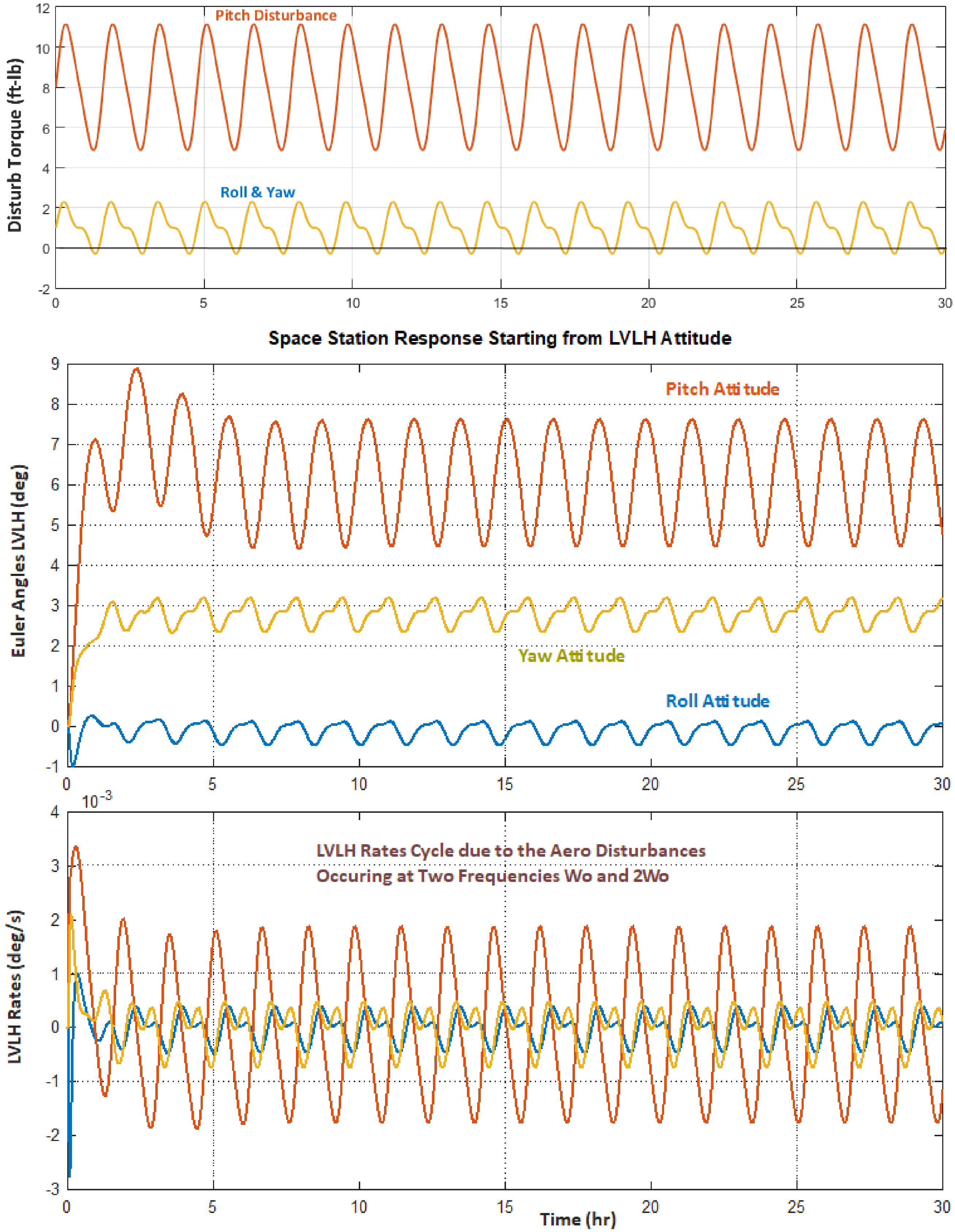


Figure 5 Spacecraft Response to Disturbances Calculated from Simulink Model "Sim_Lin_TEA_Flex.mdl"

3.4 Stability Analysis

The Simulink model "Open_Lin-TEA_Flex.mdl" is used to calculate the frequency responses of the 3 loops system by opening one loop while keeping the other two closed. The loop is opened at the CMG control torque. It is shown in Figure 6 configured for pitch analysis. The Matlab script m-file "freq.m" is using this model to calculate the Bode and Nichols plots.

Figures 7, 8 and 9 show the stability analysis results obtained from this model, one loop at a time. Structural flexibility becomes an issue if we try to open up the bandwidth too much and we can, therefore, offer only limited amount of disturbance attenuation.

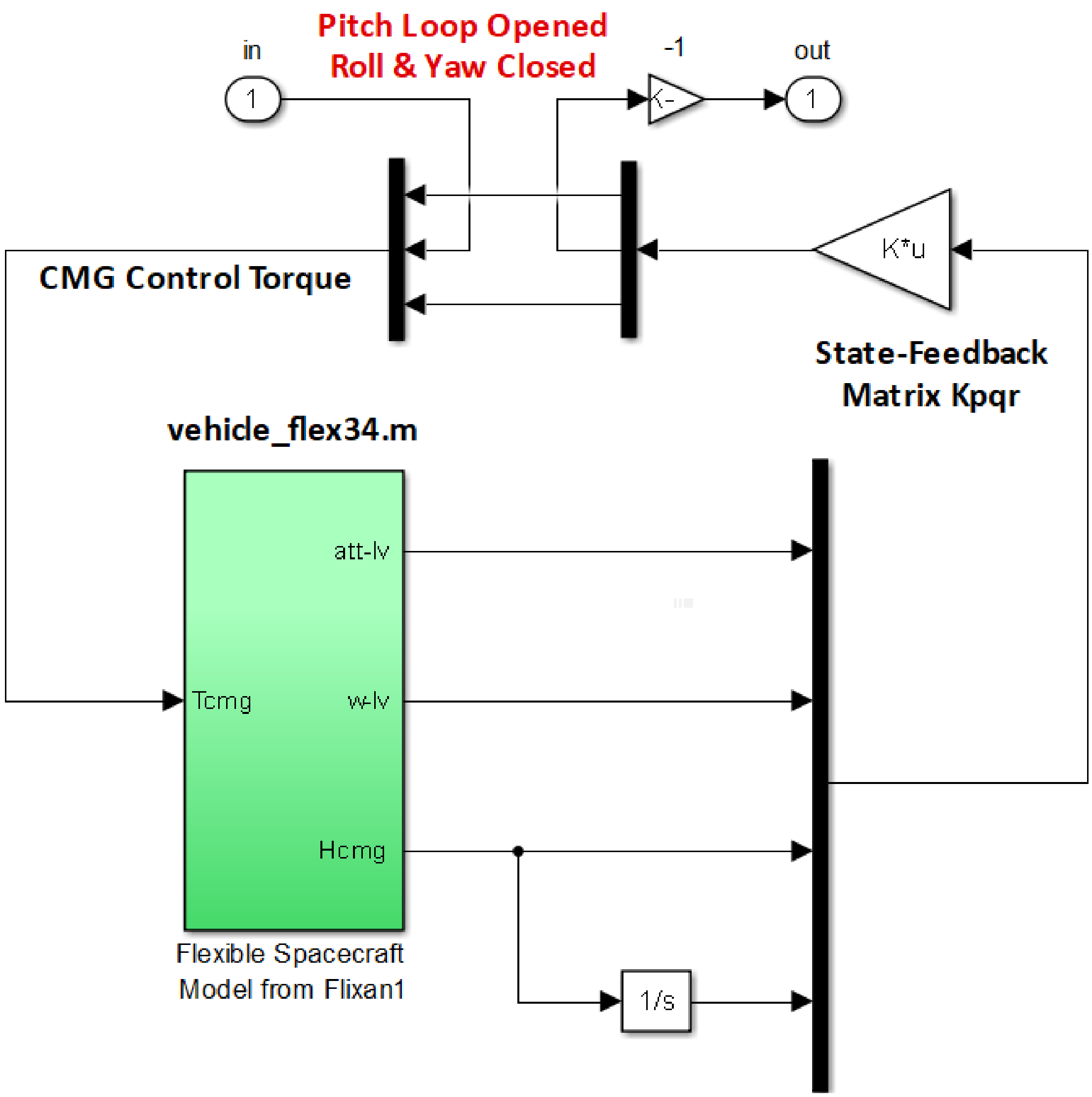


Figure 6 Simulink Model "Open_Lin_TEA_Flex.mdl" Used to Calculate the Frequency Responses and Analyze Stability

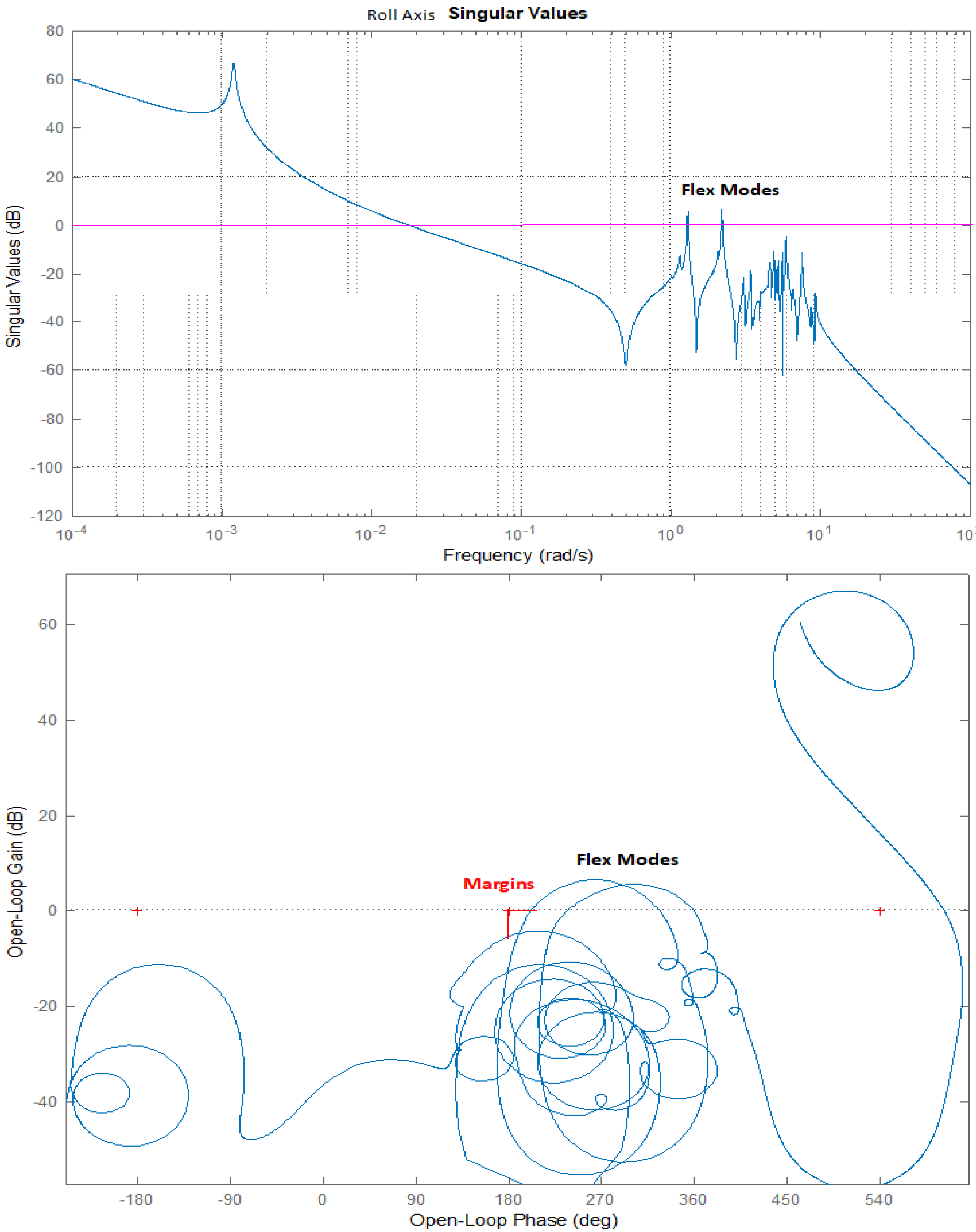


Figure 7 Roll Axis Bode and Nichols Plots showing Flexibility and Stability Margins

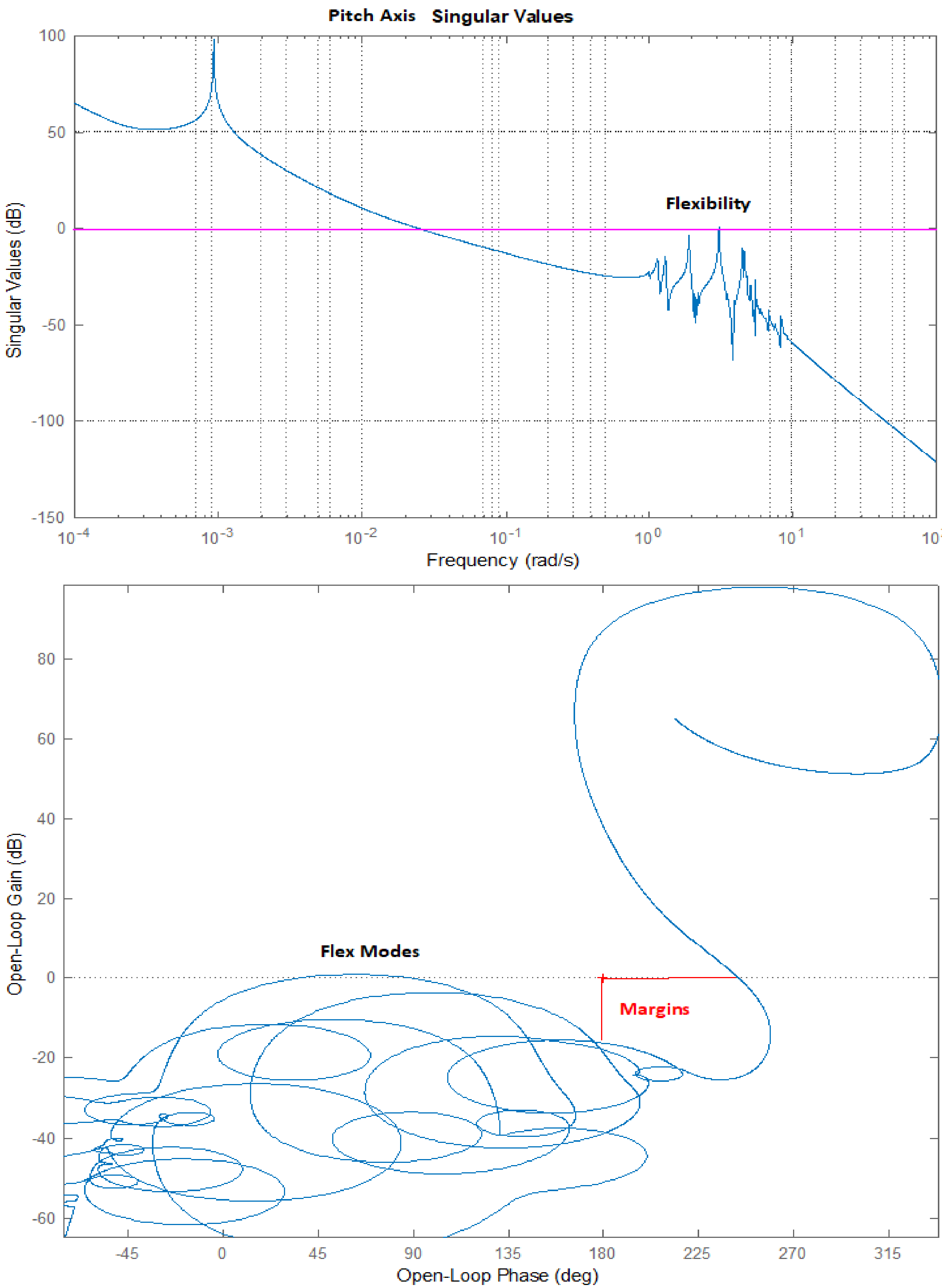


Figure 8 Pitch Axis Bode and Nichols Plots showing Flexibility and Stability Margins

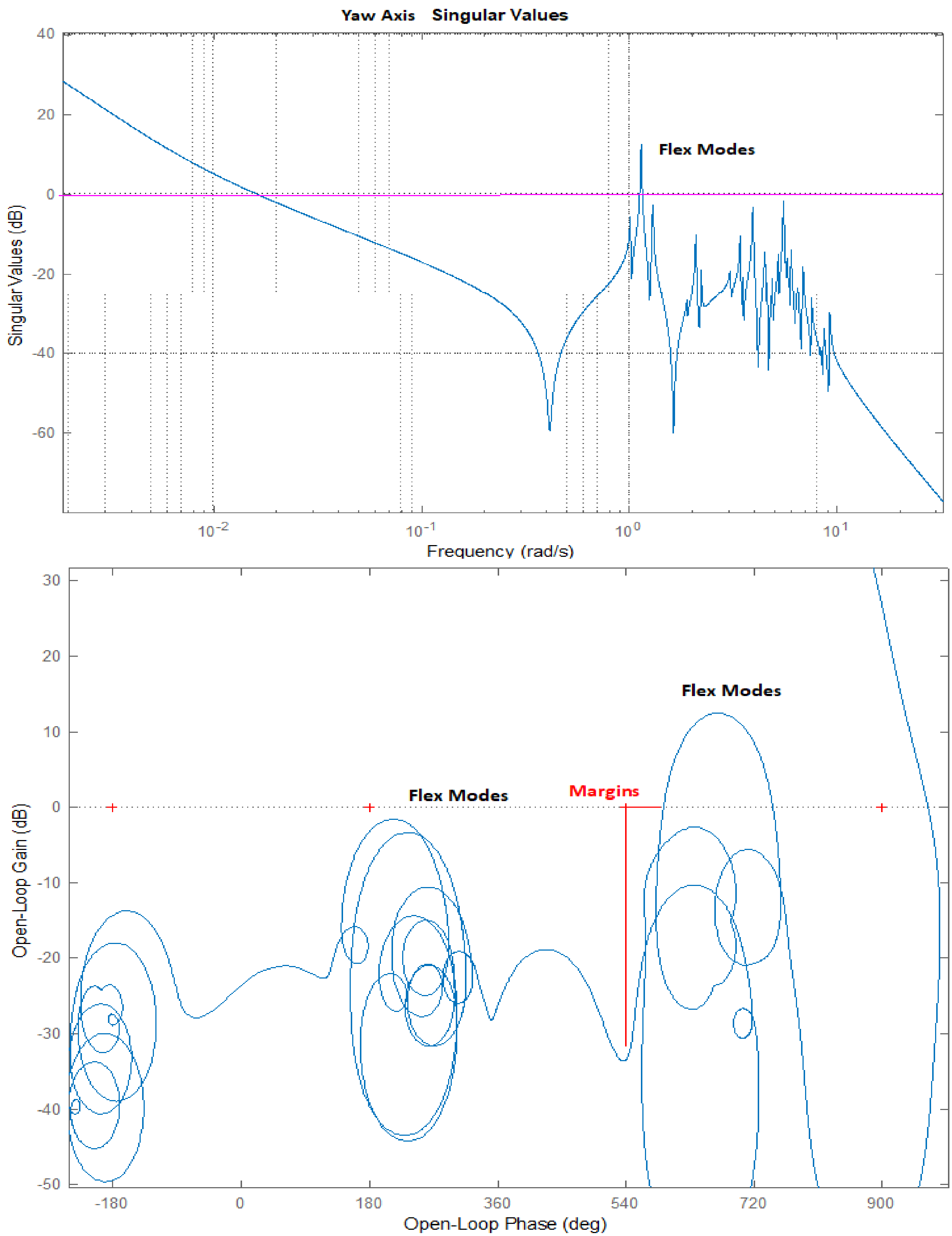


Figure 9 Yaw Axis Bode and Nichols Plots showing Flexibility and Stability Margins

4. Design with Disturbance Attenuation Filters

The cyclic disturbances occurring at orbital rate are big enough to produce sizeable attitude oscillations at orbital frequency ω_o, especially in pitch. Since we know the exact frequency of the disturbance, a more efficient design approach is to attenuate its effect on the attitude by introducing a resonance in the design system at the same frequency. The resonance is implemented by including two additional states (α_1, α_2) which are excited by the pitch attitude oscillations θ. A similar resonance with states (β_1, β_2) is introduced in yaw and it is excited by the yaw attitude oscillations ψ. The roll axis doesn't need it as much. When these states are properly penalized in the LQR optimization they will provide further attenuation in the pitch and yaw oscillations. We are essentially increasing the effectiveness of the control system at a certain frequency in order to attenuate known disturbances.

4.1 Flixan Models

The files for this analysis are in directory: *"Flixan\Control Analysis\LQG\Examples\Space-Station w CMG2\Design-2"*. The Space Station parameters are defined in the input file "*Space_Station.Inp*" which includes two flight vehicle datasets: a rigid-body "*Space Station with Double-Gimbal CMG Array (Rigid)*", and a flexible model with 34 structural modes "*Space Station with Double-Gimbal CMG Array (Flex)*". The already selected and scaled modes are in dataset "*Space Station with Double-Gimbal CMG Array, 34 Flex Modes*". The "**LVLH Attitude & Rate**" flag is included in the flags line that defines the output attitude and rate relative to the LVLH frame. The rate states however are still in body. A batch set is included at the top of the input file with title "*Batch for Large Flexible Space Station*" and it is used for processing the entire input file fast. The systems and matrices created by the program are saved in file "*Space-Station.Qdr*".

```
BATCH MODE INSTRUCTIONS ...............
Batch for Large Flexible Space Station
! This batch set creates a models for a Space Station that is
! controlled by an array of double-gimbal CMGs. Two models
! are created, a rigid-body model and a flexible model using the
! attached modal data. The design model is extracted from Rigid Vehicle
! and the plant state is transformed so that it is equal to the Output, C=I
!
Retain Matrix     : State Weight Matrix Qc (16x16)
Retain Matrix     : Control Weight Matrix Rc (3x3)
!
Flight Vehicle    : Space Station with Double-Gimbal CMG Array (Rigid)
Flight Vehicle    : Space Station with Double-Gimbal CMG Array (Flex)
System Modificat : Space Station with Double-Gimbal CMG Array (Design Plant)
System Modificat : Space Station with Double-Gimbal CMG Array (LVLH Plant)
Transf-Function   : Oscillation Filter
Transf-Function   : 3 Integrators
System Connection: Augmented Design Plant
LQR Control Des   : LQR Control Design for Augmented Space Station Model
!
To Matlab Format : Space Station with Double-Gimbal CMG Array (Rigid)
To Matlab Format : Space Station with Double-Gimbal CMG Array (Flex)
To Matlab Format : Space Station with Double-Gimbal CMG Array (Design Plant)
To Matlab Format : Space Station with Double-Gimbal CMG Array (LVLH Plant)
To Matlab Format : Augmented Design Plant
To Matlab Format : LQR State-Feedback Gain for Augmented Design Plant
----------------------------------------------------------------------------------------------------
```

```
CREATE A NEW SYSTEM FROM AN OLD SYSTEM... (Titles of the New and Old Systems)
Space Station with Double-Gimbal CMG Array (Design Plant)
Space Station with Double-Gimbal CMG Array (Rigid)
! Create the Station Design Model by Reducing the Rigid-Body Model
! Inputs are the CMG Control Torques
! Outputs are: LVLH attitude and rates, and CMG Momentum
!
TRUNCATE OR REORDER THE SYSTEM INPUTS, STATES, AND OUTPUTS
Extract Inputs :   1   2   3
Extract States :   1   3   5   2   4   6  11  12  13
Extract Outputs:   1   3   5   2   4   6  16  17  18
----------------------------------------------------------------------------------------------------
CREATE A NEW SYSTEM FROM AN OLD SYSTEM... (Titles of the New and Old Systems)
Space Station with Double-Gimbal CMG Array (LVLH Plant)
Space Station with Double-Gimbal CMG Array (Design Plant)
! Transform the design plant and make the States equal to the Outputs
SYSTEM TRANSFORMATION, STATES EQUAL TO OUTPUTS
----------------------------------------------------------------------------------------------------
```

The input file includes a system modification dataset "*Space Station with Double-Gimbal CMG Array (Design Plant)*" that creates the control design model by extracting a reduced number of inputs, states, and outputs from the rigid model. An additional transformation is needed because although the design plant attitude and rates are in the LVLH frame, some of the states, however, are still in the body frame. This transformation modifies the design plant and makes the states to be equal to the outputs which are LVLH attitudes and rates and the output matrix C= Identity. The dataset that performs this transformation is "*Space Station with Double-Gimbal CMG Array (LVLH Plant)*". Notice that the transformation requires the design plant matric C to be square and invertible.

```
SYSTEM OF TRANSFER FUNCTIONS ...
Oscillation Filter
! This Second Order Filter Amplifies the Oscillation Disturbance
! for the LQR Optimization
Continuous
TF. Block #  1  Integrator a1                          Order of Numer, Denom=  0  1
Numer 0.0         1.0
Denom 1.0         0.0
TF. Block #  2  Integrator a2                          Order of Numer, Denom=  0  1
Numer 0.0         1.0
Denom 1.0         0.0
......................................................
Block #, from Input #, Gain
      1             1       1.0
...........................
Block #, from Block #, Gain
      2             1       1.0
      1             2      -1.21e-6
...........................
Outpt #, from Block #, Gain
      1             1       1.0
      2             2       1.0
...........................
Definitions of Inputs  =   1
Pitch Attitude (rad)

Definitions of Outputs =   2
Filter Output a1
Filter Output a2
------------------------------------------------------------------------------
```

$$\dot{\alpha}_1 = \theta - \omega_o^2 \alpha_2$$
$$\dot{\alpha}_2 = \alpha_1$$

The oscillation filter is a second order resonance that is tuned at the disturbance frequency ω_0. It creates two additional states (α_1, α_2) which are used to penalize attitude oscillations at that frequency, in the LQR optimization. It is implemented as shown above by two integrators combined using the Flixan transfer functions combination utility.

```
SYSTEM OF TRANSFER FUNCTIONS ...
3 Integrators
! Used to calculate the CMG Momentum Integral for the LQR Optimization
Continuous
TF. Block #  1  Integrator x                                 Order of Numer, Denom=  0  1
Numer 0.0         1.0
Denom 1.0         0.0
TF. Block #  2  Integrator y                                 Order of Numer, Denom=  0  1
Numer 0.0         1.0
Denom 1.0         0.0
TF. Block #  3  Integrator z                                 Order of Numer, Denom=  0  1
Numer 0.0         1.0
Denom 1.0         0.0
......................................................
Block #, from Input #, Gain
      1             1       1.0
      2             2       1.0
      3             3       1.0
...........................
Outpt #, from Block #, Gain
      1             1       1.0
      2             2       1.0
      3             3       1.0
...........................
Definitions of Inputs  =   3
CMG Momentum-X
CMG Momentum-Y
CMG Momentum-Z

Definitions of Outputs =   3
CMG Momentum-X Integral
CMG Momentum-Y Integral
CMG Momentum-Z Integral
-----------------------------------------------------------------------------
```

The design model is now augmented by including 3 additional states, the (x, y, z) momentum integral in order to bound the CMG momentum from diverging and to keep it cycling near zero. Two attitude oscillation filters are also included in the augmented system to further attenuate the pitch and yaw attitude oscillations at orbital frequency. The momentum integrators are implemented as 3 transfer functions system by the transfer functions combination utility, shown above. The "*Augmented Design Plant*" is obtained by combining four systems as shown in Figure 10 using the Flixan systems combination utility. It is also implemented in the Simulink file "*Augm_Vehicle.mdl*". In addition to the 9 states of the original LVLH design plant, the augmented plant now includes the 3 momentum integral states and the four attitude filter states (2 pitch and 2 yaw).

```
INTERCONNECTION OF SYSTEMS .....
Augmented Design Plant
! Augment the Design Plant by including the Momentum Integral and the
! Oscillation Filters
Titles of Systems to be Combined
Title 1 Space Station with Double-Gimbal CMG Array (LVLH Plant)
Title 2 3 Integrators
Title 3 Oscillation Filter                                              Pitch Filter
Title 4 Oscillation Filter                                              Yaw Filter
SYSTEM INPUTS TO SUBSYSTEM  1
Via Matrix +I3                                                          3 CMG Torques
...............................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  1                                        9 Original States
System Output  1 from Subsystem  1, Output  1, Gain= 1.0                3 LVLH Attitudes
System Output  2 from Subsystem  1, Output  2, Gain= 1.0
System Output  3 from Subsystem  1, Output  3, Gain= 1.0
System Output  4 from Subsystem  1, Output  4, Gain= 1.0                3 LVLH Rates
System Output  5 from Subsystem  1, Output  5, Gain= 1.0
System Output  6 from Subsystem  1, Output  6, Gain= 1.0
System Output  7 from Subsystem  1, Output  7, Gain= 1.0                3 CMG Momentum
System Output  8 from Subsystem  1, Output  8, Gain= 1.0
System Output  9 from Subsystem  1, Output  9, Gain= 1.0
...............................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  2                                        from Moment Integr
System Output 10 from Subsystem  2, Output  1, Gain= 1.0                H-integr-X
System Output 11 from Subsystem  2, Output  2, Gain= 1.0                H-integr-Y
System Output 12 from Subsystem  2, Output  3, Gain= 1.0                H-integr-Z
...............................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  3                                        from Pitch Filter
System Output 13 from Subsystem  3, Output  1, Gain= 1.0                a1
System Output 14 from Subsystem  3, Output  2, Gain= 1.0                a2
...............................................................................
SYSTEM OUTPUTS FROM SUBSYSTEM  4                                        from Yaw Filter
System Output 15 from Subsystem  4, Output  1, Gain= 1.0                b1
System Output 16 from Subsystem  4, Output  2, Gain= 1.0                b2
...............................................................................
SUBSYSTEM NO  1 GOES TO SUBSYSTEM NO  2                                 to Momen Integrat
Subsystem  1, Output  7 to Subsystem  2, Input  1, Gain=  1.0           CMG Momentm-X
Subsystem  1, Output  8 to Subsystem  2, Input  2, Gain=  1.0           CMG Momentm-Y
Subsystem  1, Output  9 to Subsystem  2, Input  3, Gain=  1.0           CMG Momentm-Z
.......................................................................
SUBSYSTEM NO  1 GOES TO SUBSYSTEM NO  3                                 to Pitch Filter
Subsystem  1, Output  2 to Subsystem  3, Input  1, Gain=  1.0           Pitch Attitude
.......................................................................
SUBSYSTEM NO  1 GOES TO SUBSYSTEM NO  4                                 to Yaw Filter
Subsystem  1, Output  3 to Subsystem  4, Input  1, Gain=  1.0           Yaw Attitude
.......................................................................
Definitions of Inputs  =   3
Roll CMG Momentum (ft-lb)
Ptch CMG Momentum (ft-lb)
Yaw  CMG Momentum (ft-lb)

Definitions of Outputs =  16
Roll Attitude  (phi-LVLH)  (radians)
Pitch Attitude (thet-LVLH) (radians)
Yaw  Attitude  (psi-LVLH)  (radians)
Roll Rate      (p-lvlh)    (rad/sec)
Pitch Rate     (q-lvlh)    (rad/sec)
Yaw  Rate      (r-lvlh)    (rad/sec)
CMG Momentum in X-axis (ft-lb-sec)
CMG Momentum in Y-axis (ft-lb-sec)
CMG Momentum in Z-axis (ft-lb-sec)
CMG Momentum Integral X (ft-lb-sec^2)
CMG Momentum Integral Y (ft-lb-sec^2)
CMG Momentum Integral Z (ft-lb-sec^2)
Pitch Oscillation Filter Output (a1)
Pitch Oscillation Filter Output (a2)
Yaw   Oscillation Filter Output (b1)
Yaw   Oscillation Filter Output (b2)
-------------------------------------------------------------------------------
```

The above dataset contains system interconnection instructions for implementing the augmented plant model shown in Figure 10.

The next dataset included in the input file is the "*LQR Control Design for Augmented Space Station Model*" which calculates the (3x16) state-feedback matrix K_{pqr}. The state and control weight matrices Qc and Rc are already included in file "*Space_Station.Qdr*". The state-feedback matrix K_{pqr} is also saved in file "*Space_Station.Qdr*" and its title is "*LQR State-Feedback Gain for Augmented Design Plant*".

```
LINEAR QUADRATIC REGULATOR STATE-FEEDBACK CONTROL DESIGN
LQR Control Design for Augmented Space Station Model
Plant Model Used to Design the Control System from:       Augmented Design Plant
Criteria Optimization Output is Matrix C
State Penalty Weight (Qc) is Matrix:   Qc16               State Weight Matrix Qc (16x16)
Control Penalty Weight (Rc) is Matrix: Rc3                Control Weight Matrix Rc (3x3)
Continuous LQR Solution Using Assymptotic Method
LQR State-Feedback Control Gain Matrix Kpqr               LQR State-Feedback Gain for Augmented Design Plant
--------------------------------------------------------------------------------------------------------------
```

Six Matlab conversion datasets are also included that convert the systems and matrices in an m-file format that can be loaded into Matlab by running the script file start.m. The files "vehicle_rigid.m" and "vehicle_flex34.m" are used in simulations. The augmented plant "augm_plant.m" can also be used in Matlab to calculate the state-feedback matrix K_{pqr}.

```
CONVERT TO MATLAB FORMAT ........      (Title, System/Matrix, m-filename)
Space Station with Double-Gimbal CMG Array (Rigid)
System
vehicle_rigid
-----------------------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........      (Title, System/Matrix, m-filename)
Space Station with Double-Gimbal CMG Array (Flex)
System
vehicle_flex34
-----------------------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........      (Title, System/Matrix, m-filename)
Space Station with Double-Gimbal CMG Array (Design Plant)
System
design_plant
-----------------------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........      (Title, System/Matrix, m-filename)
Space Station with Double-Gimbal CMG Array (LVLH Plant)
System
lvlh_plant
-----------------------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........      (Title, System/Matrix, m-filename)
Augmented Design Plant
System
augm_plant
-----------------------------------------------------------------------------------------------------
CONVERT TO MATLAB FORMAT ........      (Title, System/Matrix, m-filename)
LQR State-Feedback Gain for Augmented Design Plant
Matrix Kpqr
-----------------------------------------------------------------------------------------------------
```

4.2 LQR Control Design with Filters

The initialization file "*start.m*" loads the dynamic systems and matrices for the simulation and analysis models. In addition to Flixan, the Matlab script file "*des.m*" can also calculate the LQR control gains. It is using the augmented design plant or the Simulink model "*Augm_Vehicle.mdl*" shown in Figure 10, which includes the spacecraft LVLH dynamics of Equation 1, the momentum integral states, and the two attitude filters. The state vector of the augmented design plant includes the 4 filter states (α_1, α_2, β_1, β_2) and the augmented 16-state system is now applied in the LQR optimization algorithm. The disturbance accommodation filters will penalize the attitude oscillations at orbital rate and ultimately provide further attenuation at that frequency.

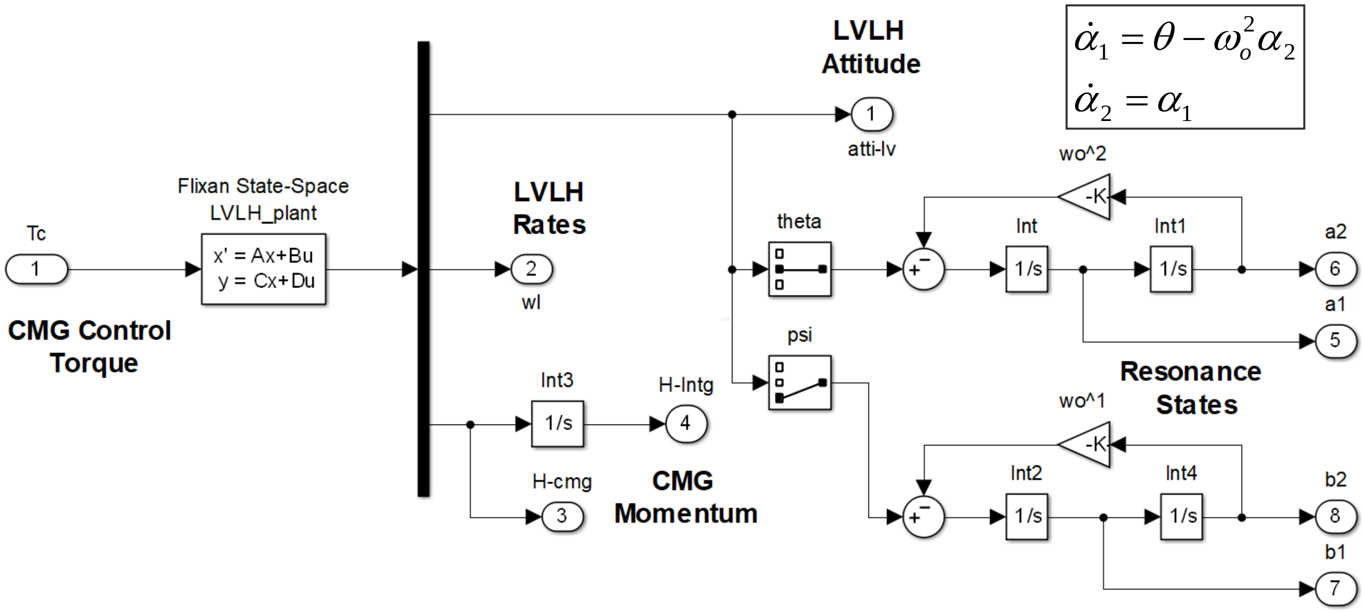


Figure 10 Augmented Design Plant "Augm_Vehicle" Consists of Equations 1, Momentum Integrals and the Pitch and Yaw Attitude Augmentation States (α_1, α_2) and (β_1, β_2)

Figure 11 shows the simulation model "*Sim_Lin_TEA-Flex.mdl*" that includes the Flixan generated system "vehicle_flex34.m" with 34 flex modes. It includes also the two second order attitude augmentation filters which are now part of the control system and produce the 4 additional states: (α_1, α_2, β_1, β_2) required for state-feedback. The rate gyro measurements from the dynamic model are body rates and they measure structural flexibility. A transformation is used to convert the rate signals to LVLH rates as expected for feedback. The spacecraft model also includes the CMG dynamics implemented as second order transfer functions and the aerodynamic disturbances, as in Fig.4.

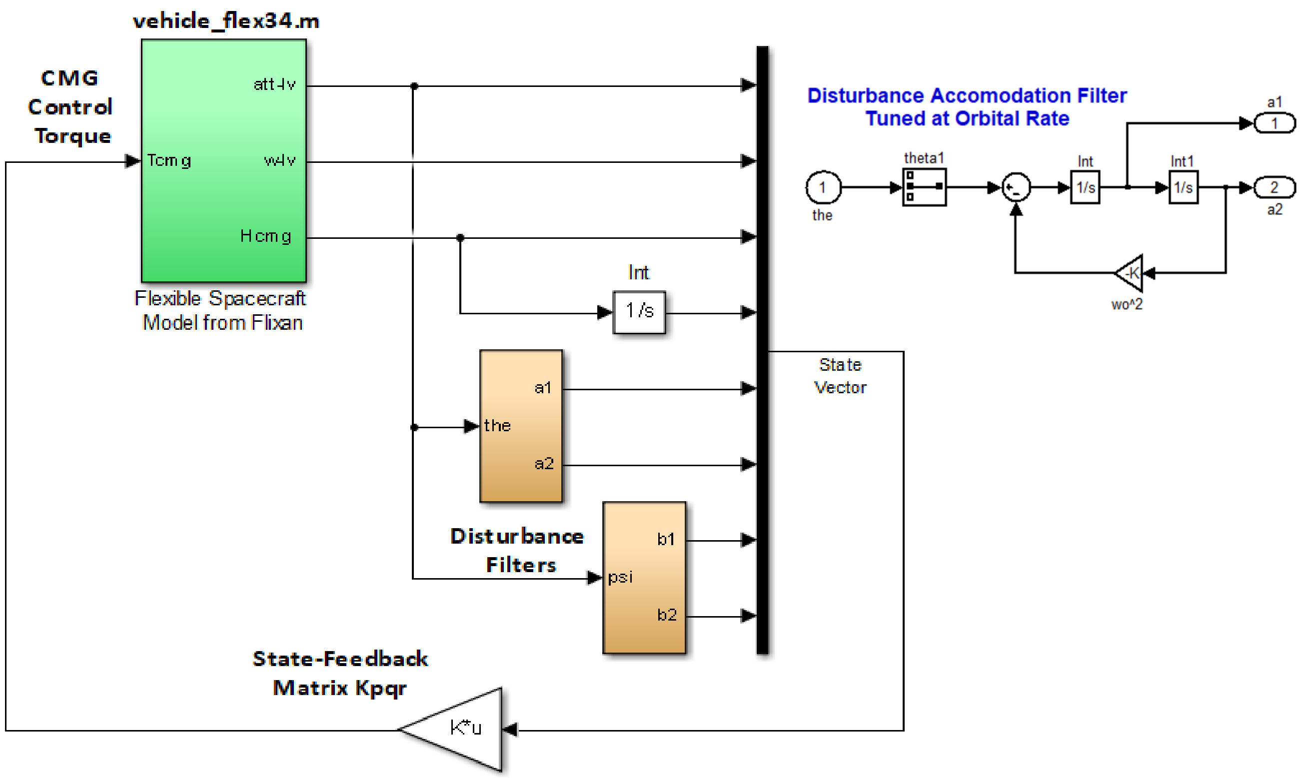
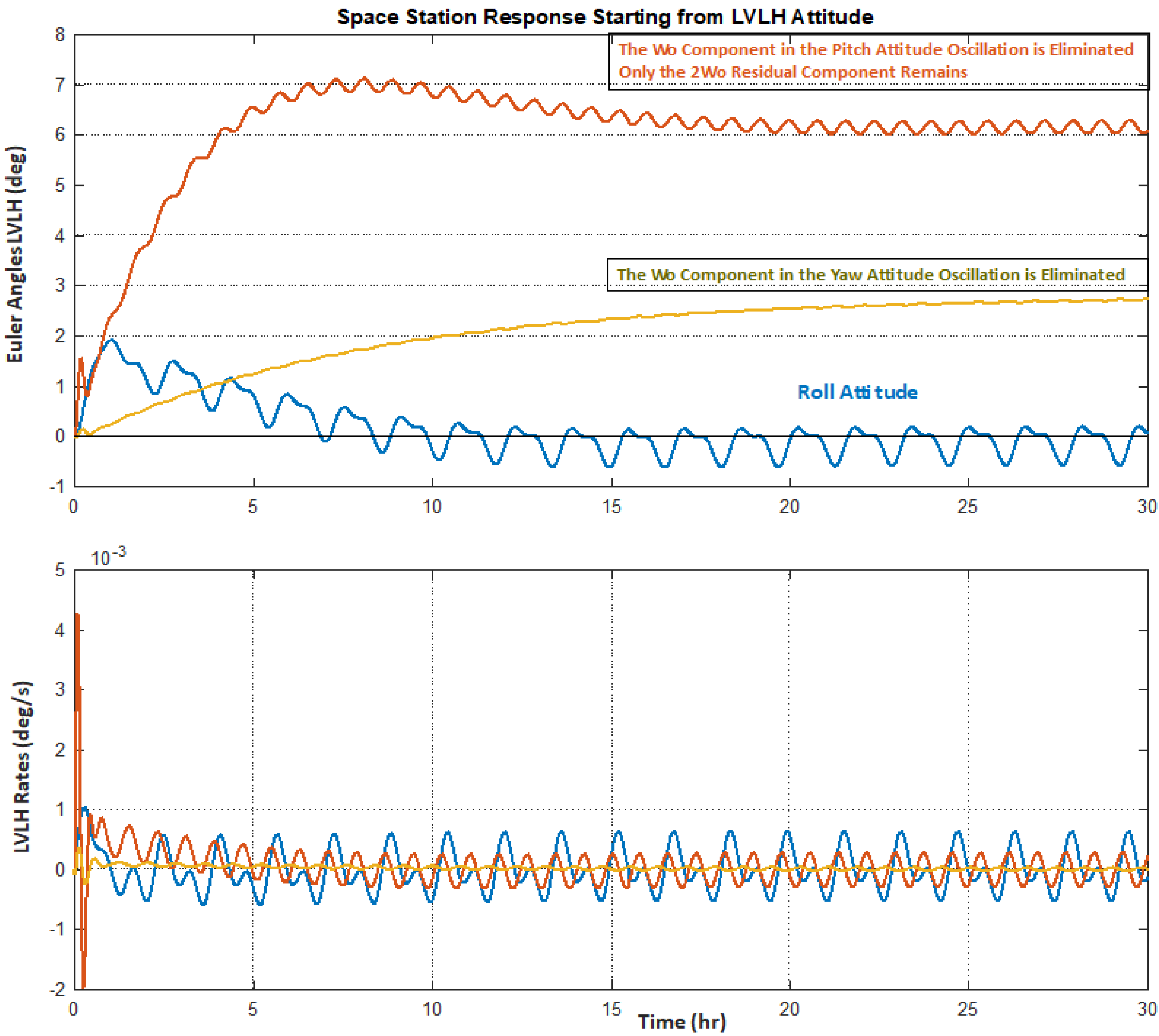


Figure 11 Flex Simulation Model "Sim_Lin_TEA-Flex.mdl"

4.3 Simulation Results

The simulation results in Figure 12 show the Space Station response to the aerodynamic disturbances as its attitude drifts towards the TEA where the external torques balance in all directions. At steady-state the pitch attitude converges to 6.3° and the yaw attitude to 3° relative to the LVLH frame. The roll attitude is small at -0.5°. The CMG momentum does not diverge but it oscillates about zero without bias as the CMGs are supplying torque to counteract the cyclic disturbances. The CMG torque is also centered at around zero. The augmented state-feedback not only stabilizes the spacecraft but it also attenuates the ω_o attitude oscillations in pitch and yaw and produces a closed loop system that is resisting the disturbances at orbital rate without the need to increase the control system bandwidth. In pitch the only remaining oscillation is due to the aero disturbance at $2\omega_o$. The yaw attitude is almost perfectly clean from oscillations. Additional filters can be included to attenuate the $2\omega_o$ frequency but we leave this as an exercise for the reader.



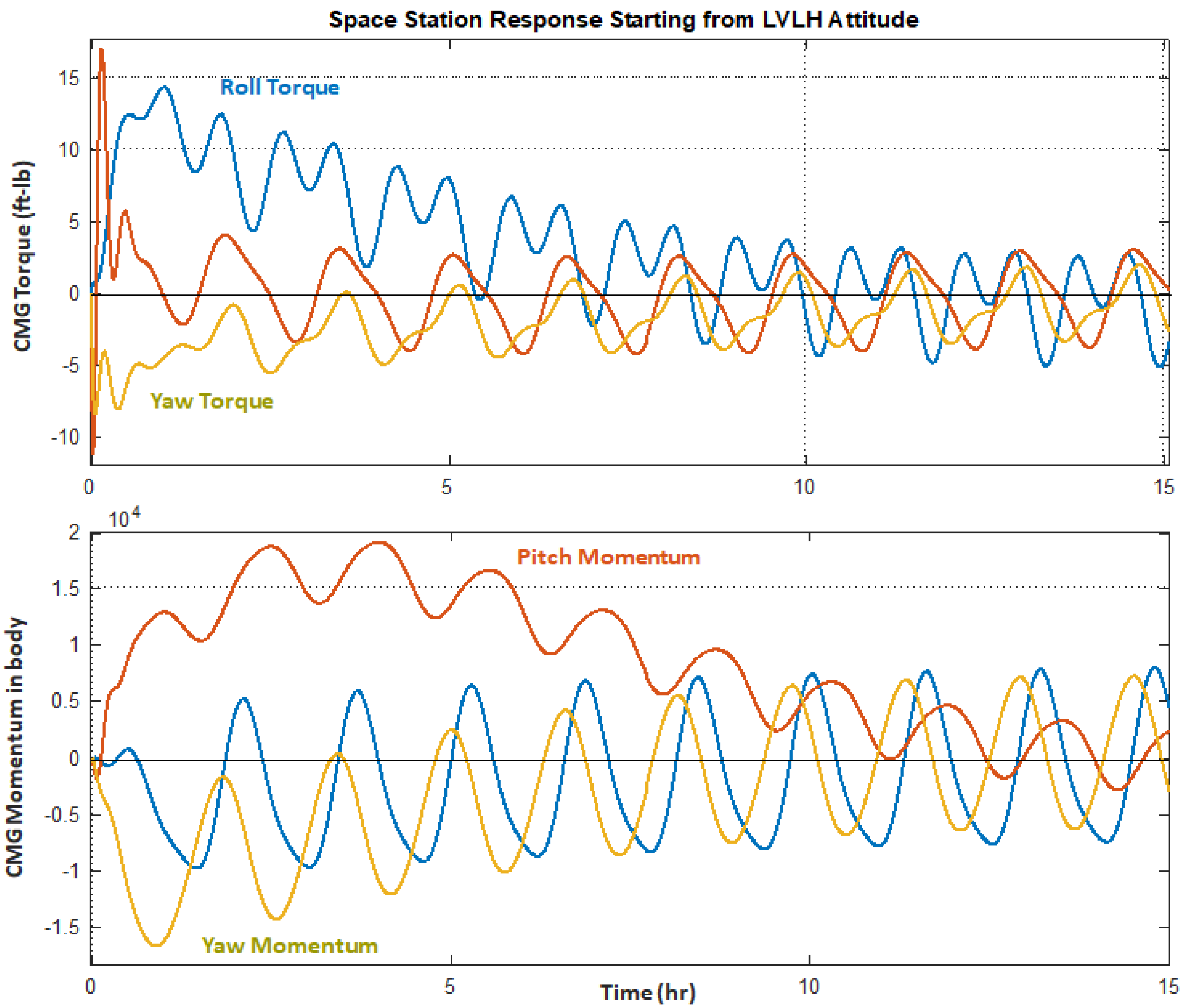


Figure 12 Spacecraft Response to Disturbances from Simulink Model "Sim_Lin_TEA_Flex.mdl"

4.4 Stability Analysis

The Simulink model "Open_Lin_TEA_Flex.mdl" is used to calculate the frequency responses of the 3 loops system by opening one loop while keeping the other two closed. The loop is opened at the CMG control torque. It is shown in Figure 13 configured for roll axis analysis. The Matlab script m-file "freq.m" is using this model to calculate the Bode and Nichols plots.

Figures 14, 15 and 16 show the stability analysis results obtained from the open-loop model, one loop opened at a time. This system has a low bandwidth of $10\omega_0$. Notice that the filters introduced a big resonance at orbital frequency ω_0=0.0011 (rad/sec) which increases the gain of the system only at that frequency. The increased magnitude is needed in order to provide the torque that will counteract the disturbance at that frequency without increasing the overall system gain that would otherwise amplify the flex modes to instability.

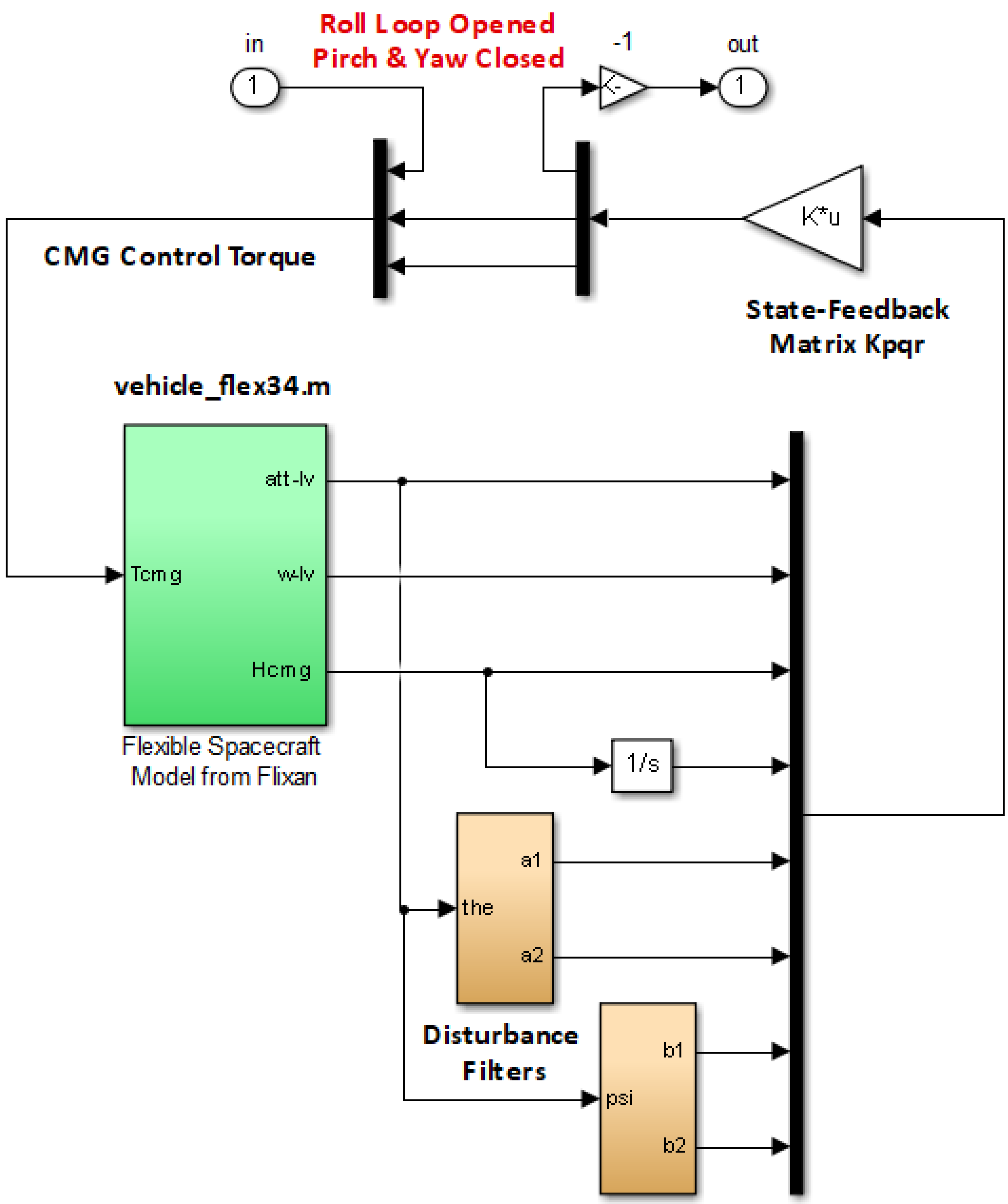


Figure 13 Simulink Model "Open_Lin_TEA_Flex.mdl" Used to Calculate the Frequency Responses and Analyze Stability

5. Conclusion

The LQR method was used to design state-feedback control system for the space station that uses gravity gradient to prevent the CMG momentum from saturating but it is cycling at around zero while it provides the torque necessary to counteract aerodynamic disturbances. The spacecraft attitude is not commanded but it drifts and slightly oscillates about the TEA. Filters are used to amplify the system's response at the disturbance frequency, counteract the disturbance effect, and to minimize the attitude oscillations.

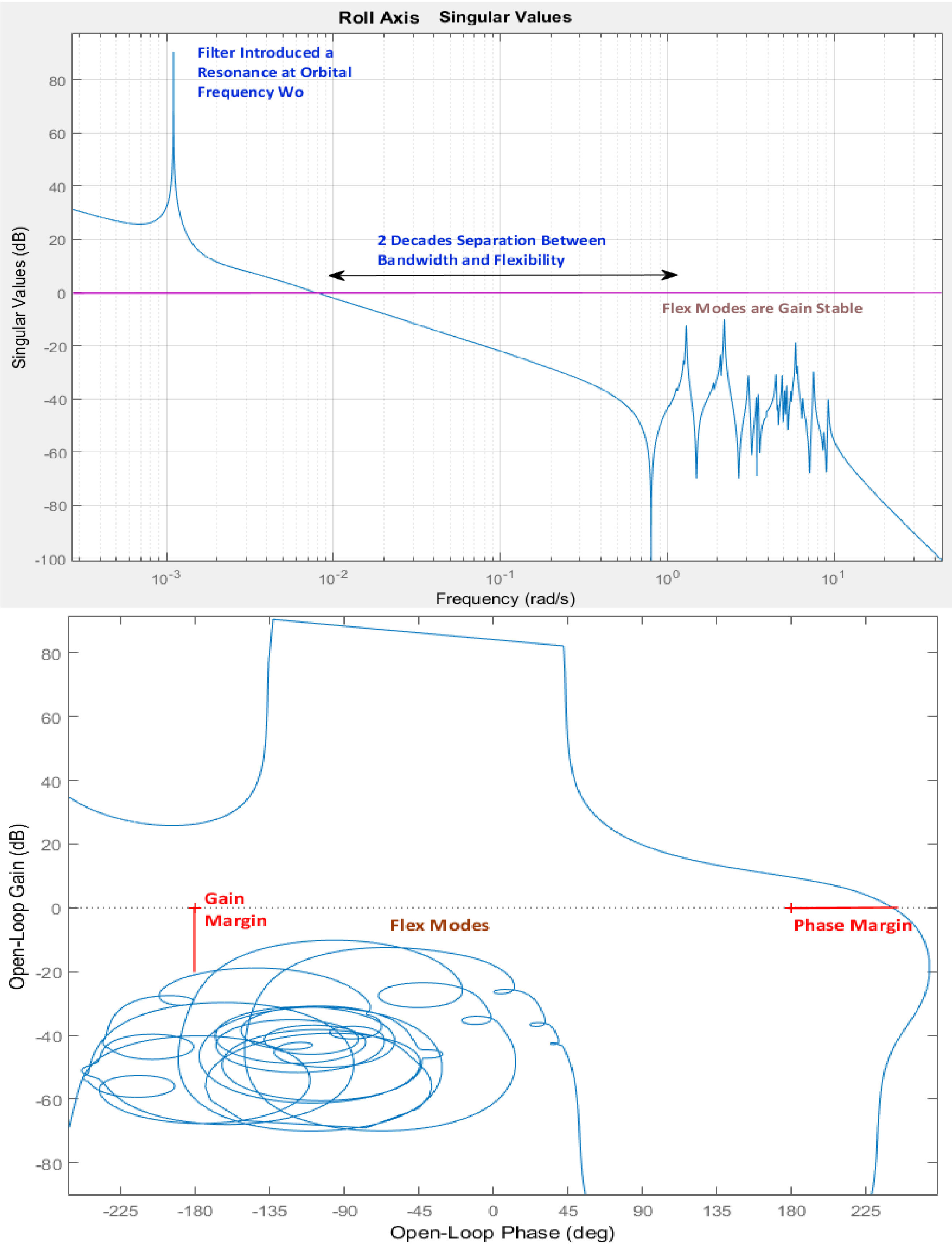


Figure 14 Roll Axis Bode and Nichols Plots showing Flexibility and Stability Margins

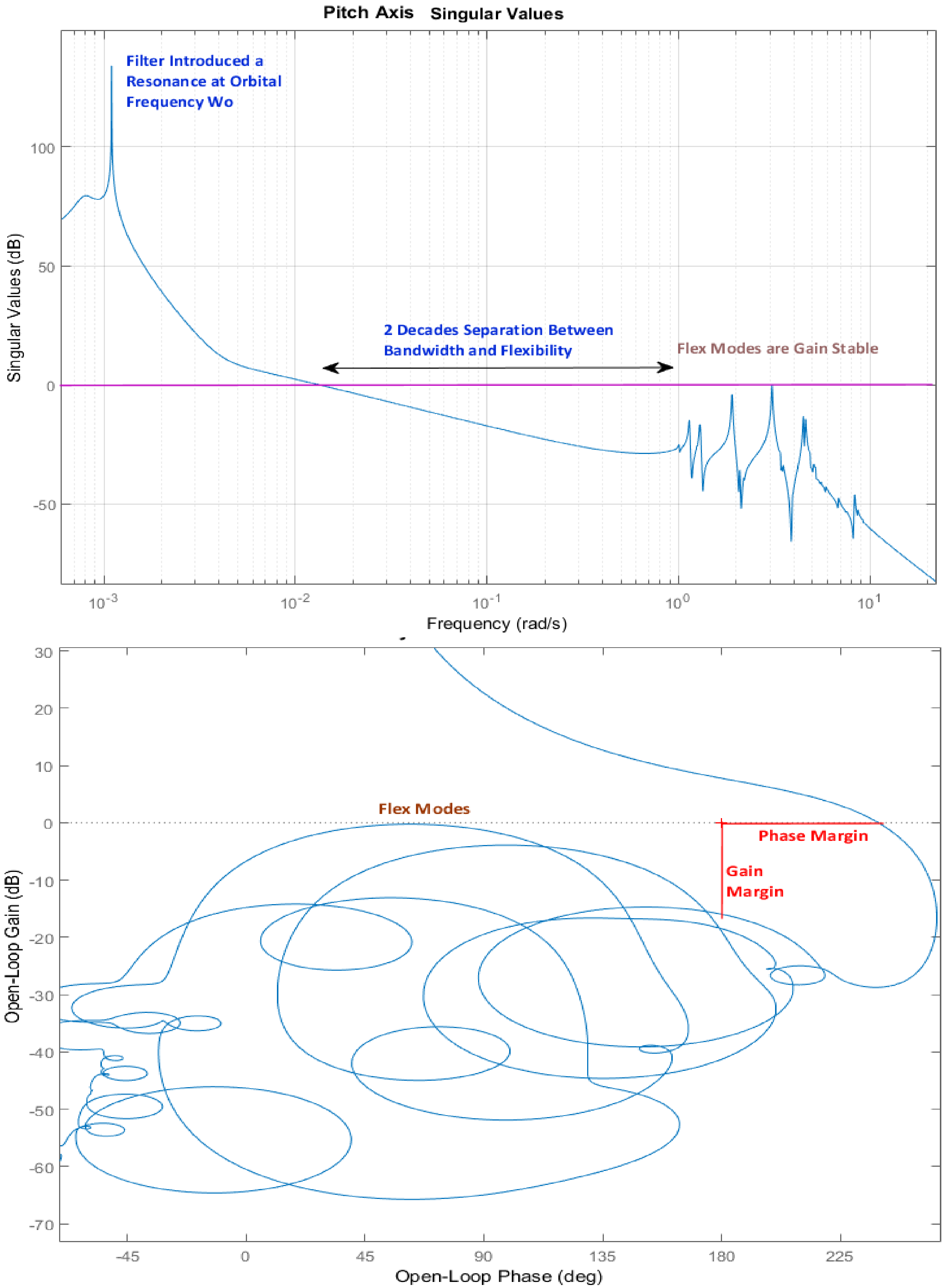


Figure 15 Pitch Axis Bode and Nichols Plots showing Flexibility and Stability Margins

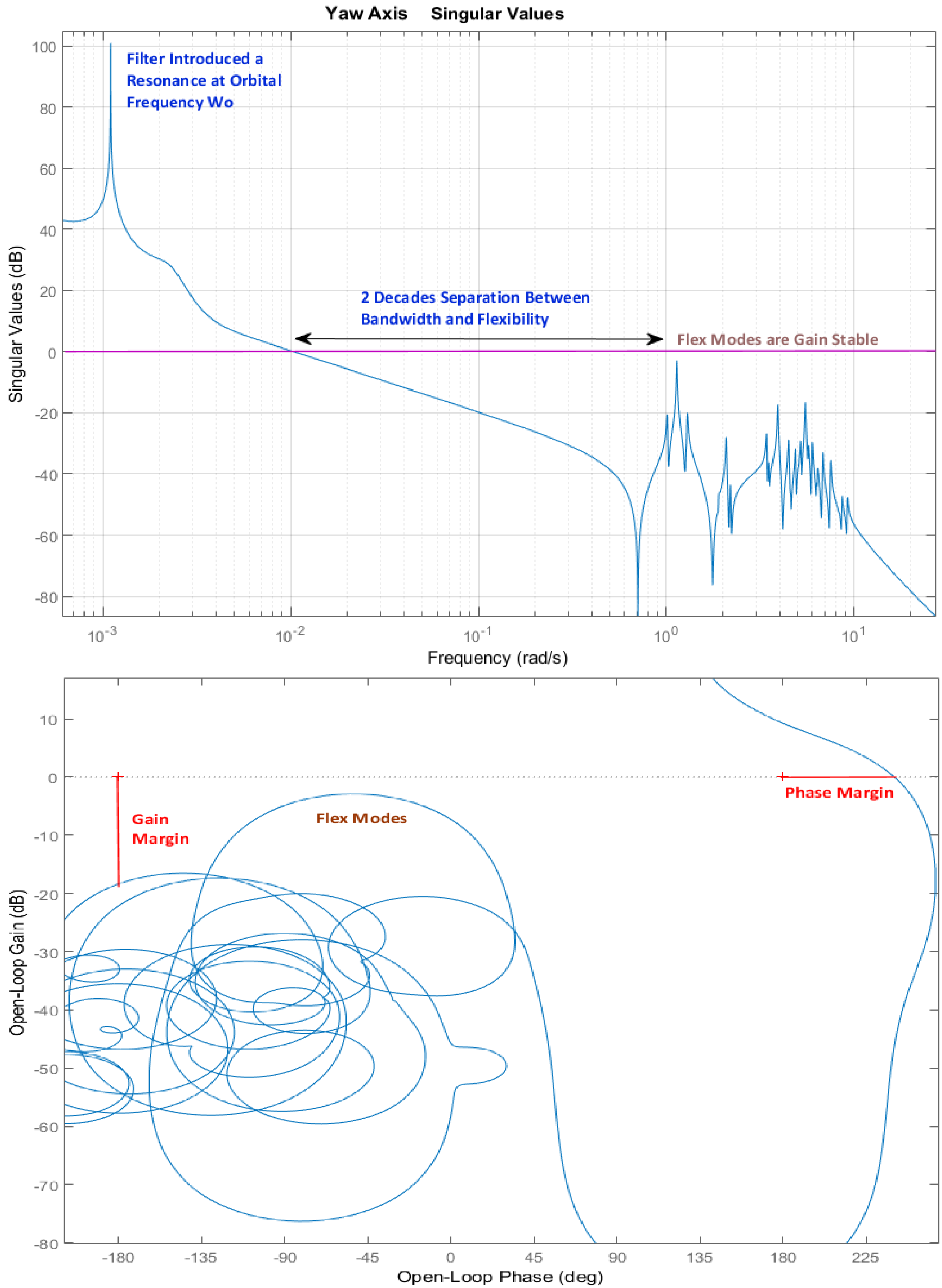


Figure 16 Yaw Axis Bode and Nichols Plots showing Flexibility and Stability Margins